Application of Particle Image Velocimetry in Aeroacoustics

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Overview

- Motivation
- Basics of PIV ("Particle Image Velocimetry")
- Combination of PIV with microphone measurements
- Results
  - First example: Flow about a circular cylinder (aeolian tone)
  - Second example: Rod-airfoil interaction
  - Third example: Slat noise
- Concluding remarks

Motivation

- Goal of aeroacoustic experiments
  - Localization and quantification of sources
  - Understanding of source mechanisms
  - Validation of numerical results
- Measurements in the far field
  - Techniques: Acoustic mirror, microphone array
  - Application in wind tunnels (also in closed test sections)
  - Far field data give usually no or only little information about source mechanisms.
  - For the understanding of the source process additional information from the source region (flow field) is required: Numerical simulations, measurements in the near field, source model

Motivation

- Measurements in the source region
  - Source strength of an acoustic analogy obtained from measurements; Example: Lighthills acoustic analogy:

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2\,\Delta\right)\rho' = \frac{\partial T_{ij}}{\partial x_i \partial x_j}$$

$$T_{ij} = \rho v_i v_j + \tau_{ij} - \delta_{ij}\left\{p' - c_0^2 \rho'\right\}$$

  - Often: $T_{ij} \approx \rho_0 v_i v_j$
  - In practice a source strength distribution is difficult to interpret
    - Large portion of the source strength does not radiate to the far field (only reactive power)
  - Integration of source strength requires high accuracy
    - Boundary effects, incomplete superpositions

Motivation

- Combination of source region and far field measurements
  - Synchronized measurement
  - Correlation of source strength with far-field pressure
  - Gives directly idea of which part of the flow field is involved in sound generation
  - Direction dependent analysis of the source process
  - Source strength which does not radiate into the far field is not visible
  - Correlation results can be used for validation of numerical simulations

Motivation

- Correlation of source strength with far-field pressure
  - Literature

    Clark und Ribner (1969), airfoil, lift

    Lee und Ribner (1972), jet, velocity (hot wire)

    Siddon (1973), flat disk, surface pressure

    Meecham und Hurdle (1974), jet, near-field pressure

    Schaffer (1979), jet, velocity (LDV)

    Panda et al. (2005), jet, Lighthill stress $\rho v_i v_j$ (Rayleigh-Scattering)
- "Causality correlation"
  - "Probe Contamination" by hot wire or pressure probe
  - Result: Source strength correlates not better that other quantities

Question: Can PIV ("Particle Image Velocimetry") be used for causality correlation ?

Motivation

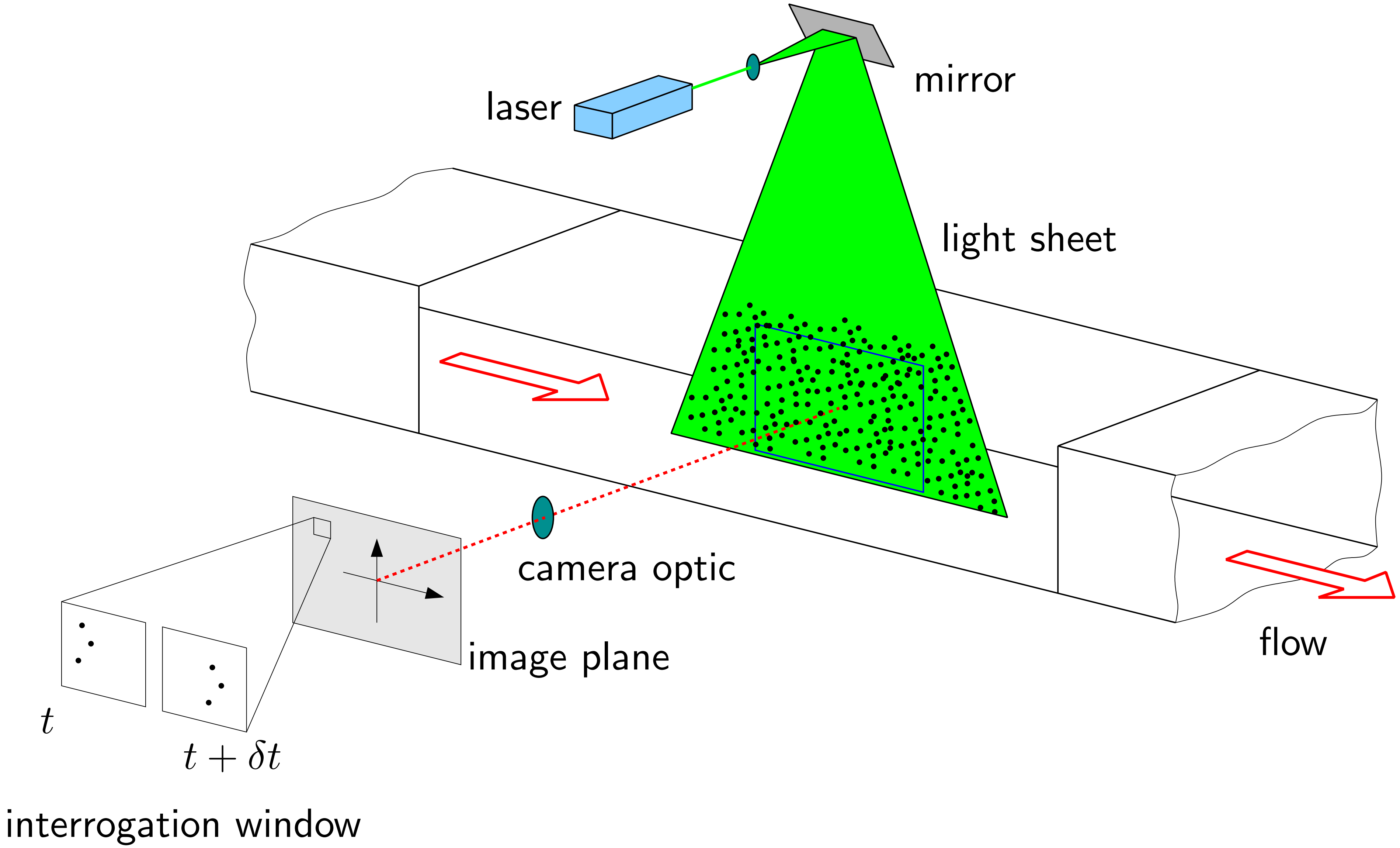
- PIV ("Particle Image Velocimetry")
  - Non-intrusive laseroptical method
  - Gives complete velocity field in one plane (planar PIV)
  - Detection of structures in the source region which correlate with the far field
  - Powell/Howe

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2\,\Delta\right)\rho' = \rho_0\,\mathrm{div}(\boldsymbol{\omega} \times \mathbf{v})$$

Vorticity $\boldsymbol{\omega}$ important quantity in source term;
Planar PIV gives one component of $\boldsymbol{\omega}$ (ω_z)

☐ First step: Feasibility test

Illustration of PIV



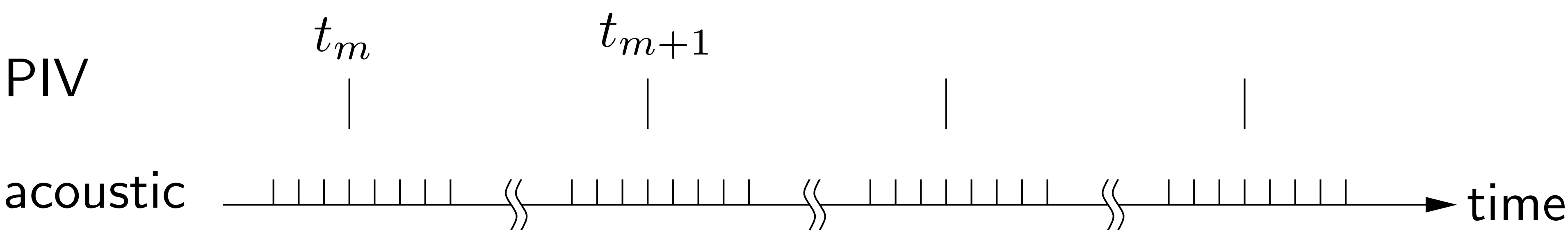
PIV in wind tunnels

- Standard system
  - Nd:YAG puls laser (double puls with 10 Hz, 2 × 180 mJ)
  - CCD-Camera with double-frame capability (PCO-Sensicam, 1280 Pixel × 1024 Pixel, camera dead-time 400 ms)
  - Seeding: Small droplets (1 μm)
    - Olive oil, DEHS (Di-Ethyl-Hexyl-Sebacat)
    - Seeding generator, seeding in settling chamber

- No analog out like for example from hot wire
  - Relativ low sampling rate
  - No anti-alias filter
  - No direct calculation of correlation

Synchronized measurement

- PIV system is controlled by programmable pulse generator
- Puls generator starts acoustic data acquisition (102.4 kHz)
- "Q-Switch"-signal of first laser is recorded
  - Assignment of PIV snapshots to pressure samples



$u(\mathbf{x}, t_m), v(\mathbf{x}, t_m)$ $\quad t_{m+1} - t_m = \Delta t_{\mathrm{piv}} \approx 400\,\mathrm{ms}$

$m = 1, ..., L$

$p'(t_n)$ $\quad t_{n+1} - t_n = \Delta t_{\mathrm{a}} \approx 10\,\mu\mathrm{s}$

Calculation of the cross correlation

- Fluctuations

$$p(t) = \bar{p} + p'(t) \ , \ \ u(\mathbf{x},t) = \bar{u}(\mathbf{x}) + u'(\mathbf{x},t)$$

- Cross correlation

$$R_{u'p'}(\mathbf{x},\tau) = \langle u'(\mathbf{x},t)\, p'(t+\tau)\rangle$$

- Discrete cross correlation

$$\tau = k\Delta t_{\mathrm{a}} \ , \ \ k \in \mathbb{N}$$

$$R_{u'p'}(\mathbf{x},k\Delta t_a) = \frac{1}{L}\sum_{m=1}^{L} u'(\mathbf{x},t_m)\, p'(t_m + k\Delta t_a)$$

- Typical parameters

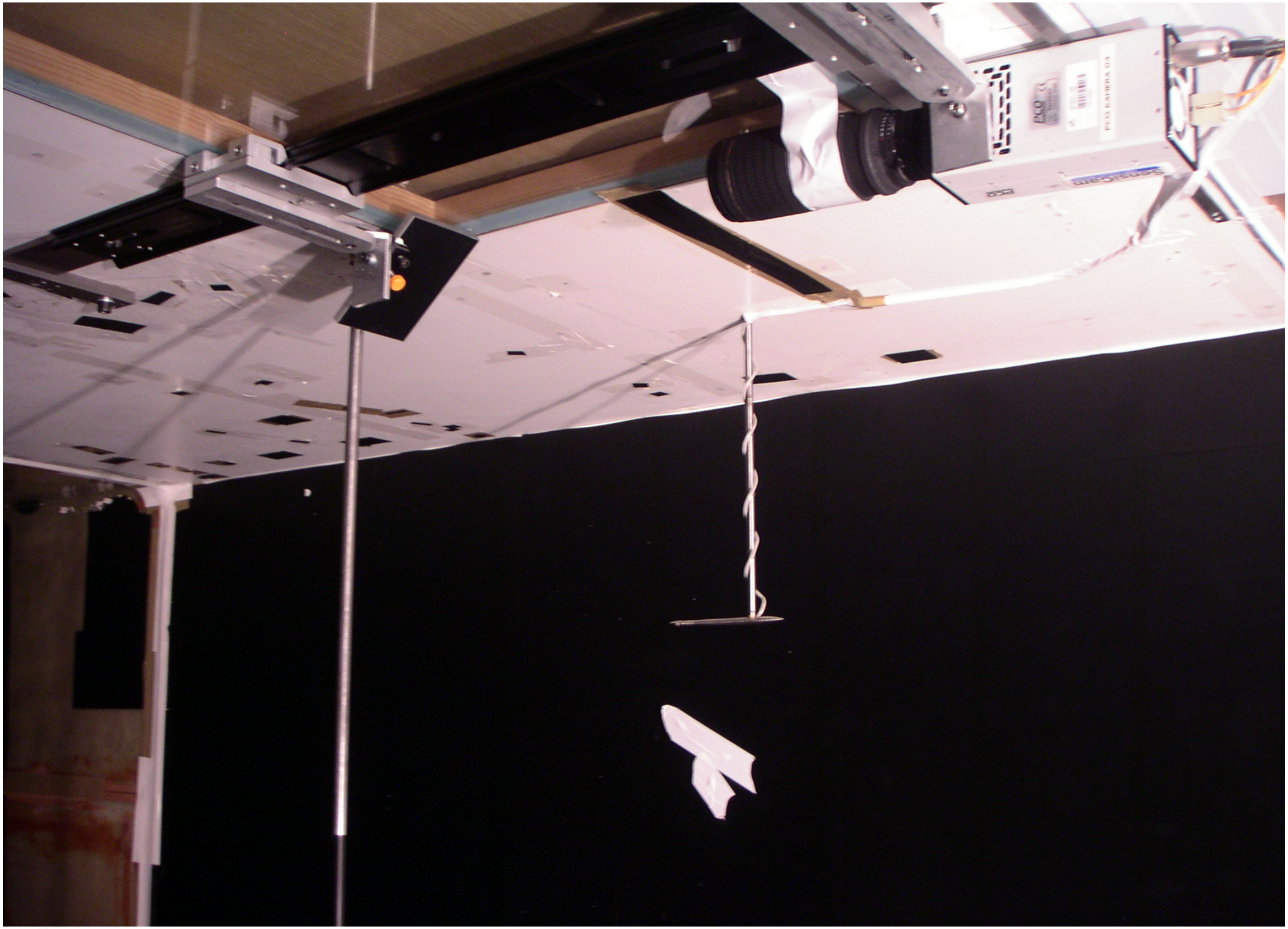
$L = 5000$, -0.2 s $\leq k\,\Delta t_{\mathrm{a}} \leq$ 0.2 s , $-20000 \leq k \leq 20000$

First example: Circular cylinder

- Flow around a circular cylinder (Henning et al., Exp. in Fluids 2008)
- Free-stream velocity $U = 20\,\mathrm{m/s}$, diameter $d = 14\,\mathrm{mm}$, Reynolds number $Re = 19000$
  - strong tonal sound (aeolian tone)
- PIV measurement
  - Light sheet perpendicular to cylinder
  - Lens: 60 mm focal length (view space 121 mm $\times$ 96 mm)
  - Frame rate (double images) $f_{\mathrm{piv}} = 2.5\,\mathrm{Hz}$, $\delta t = 30\,\mu\mathrm{s}$
- Far-field measurement
  - Microphone 1/4" (B&K), nose cone
  - Sampling rate $f_{\mathrm{a}} = 102.4\,\mathrm{kHz}$

Experimental setup

Experimental setup



Experimental setup

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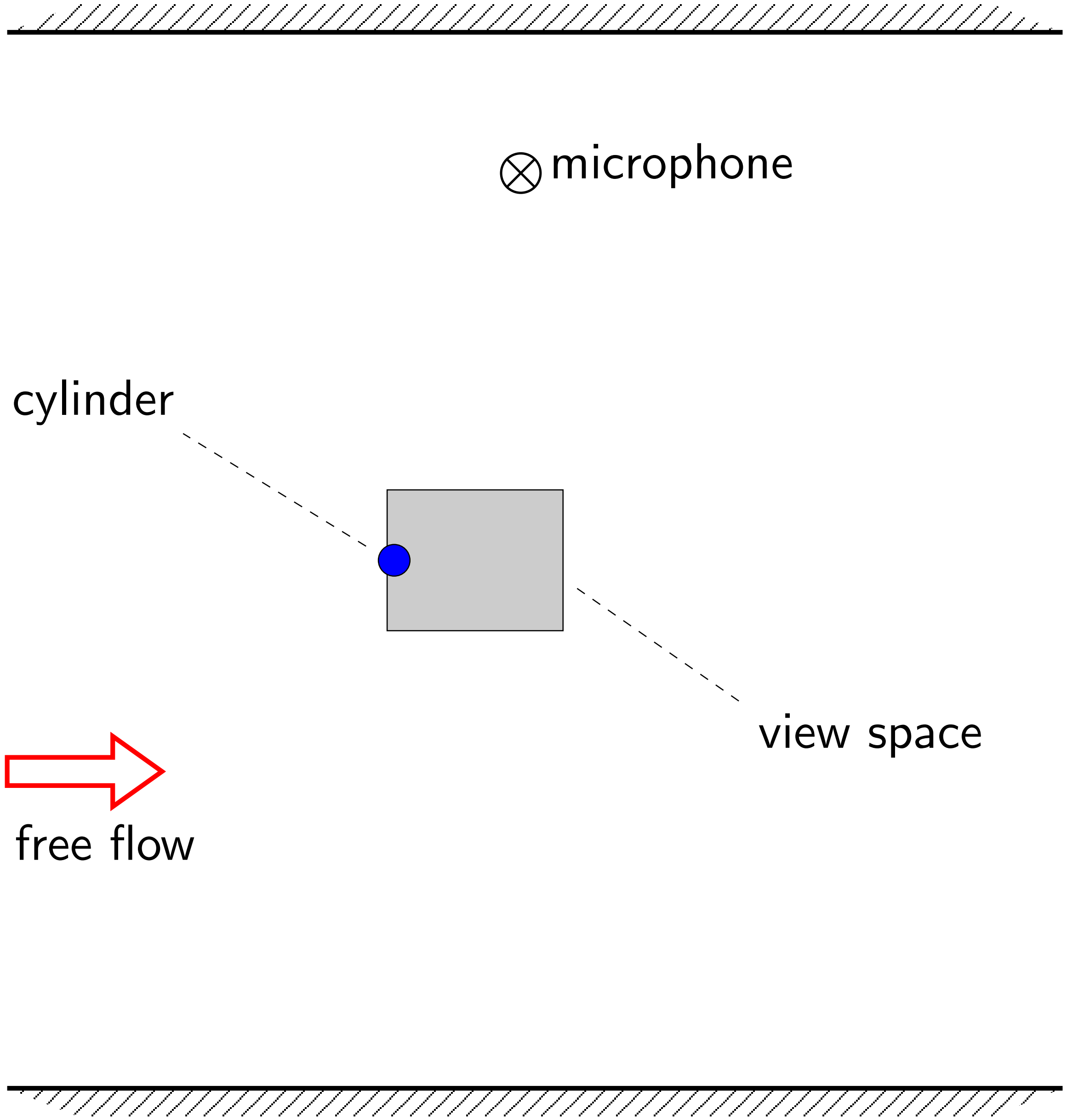
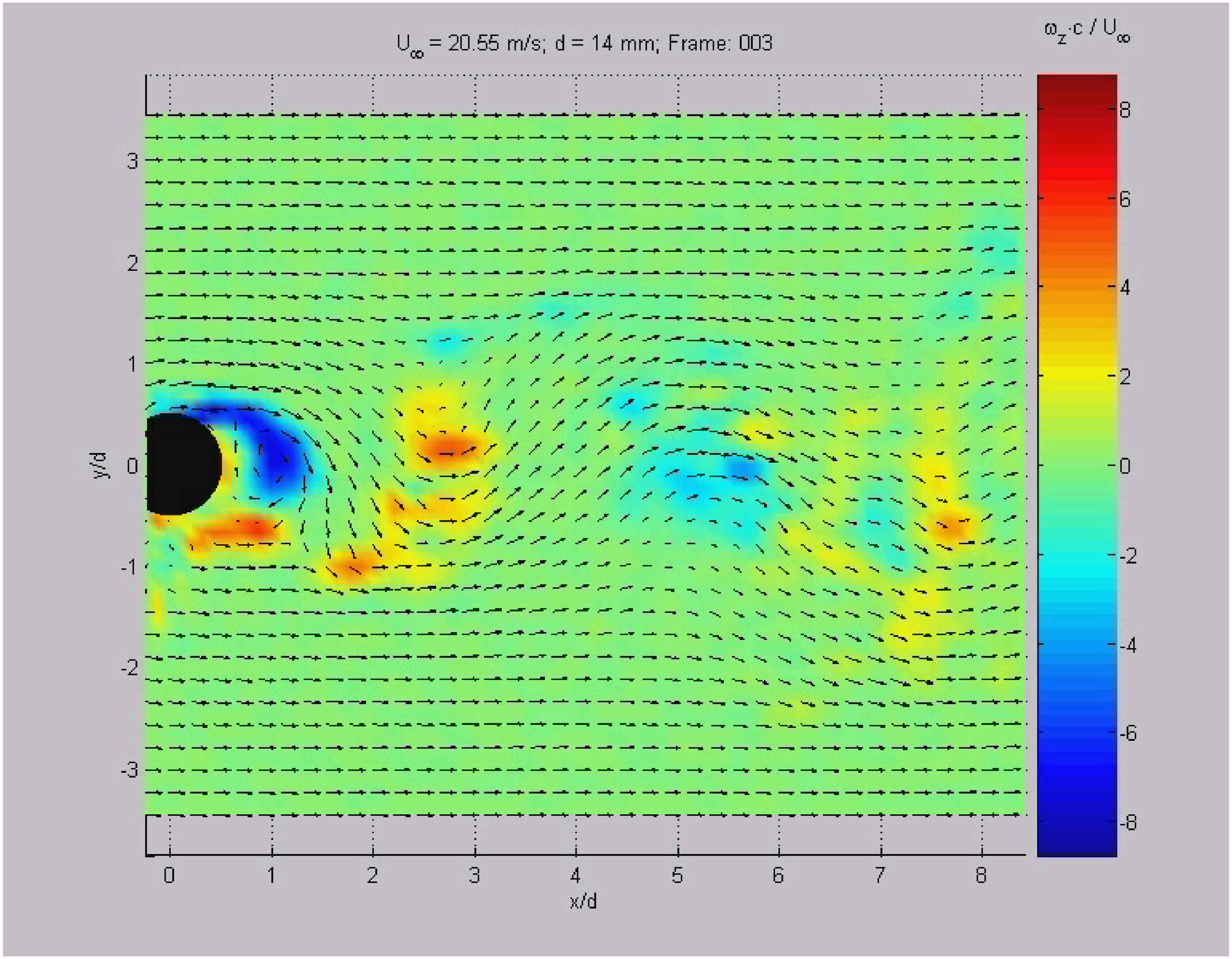
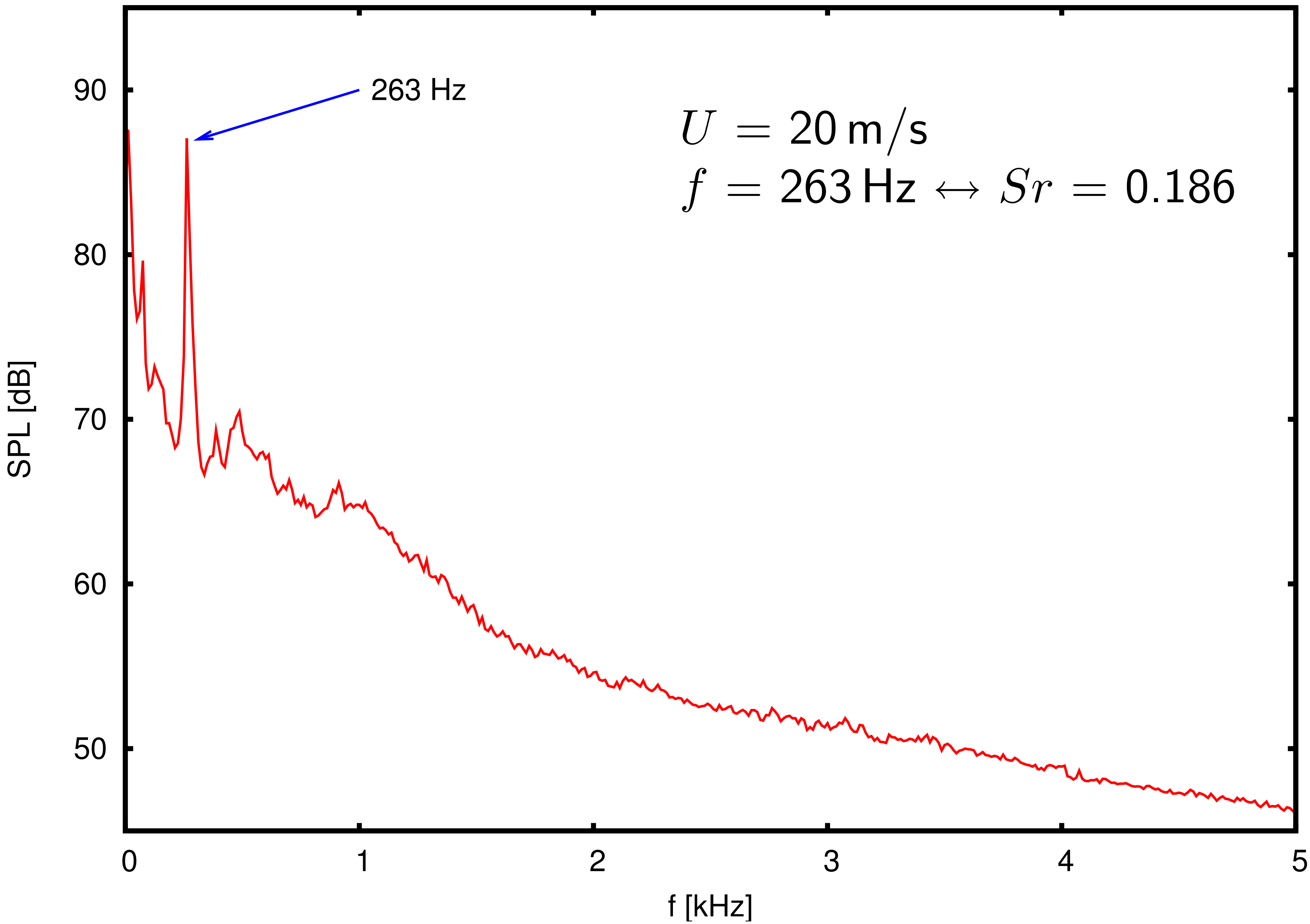


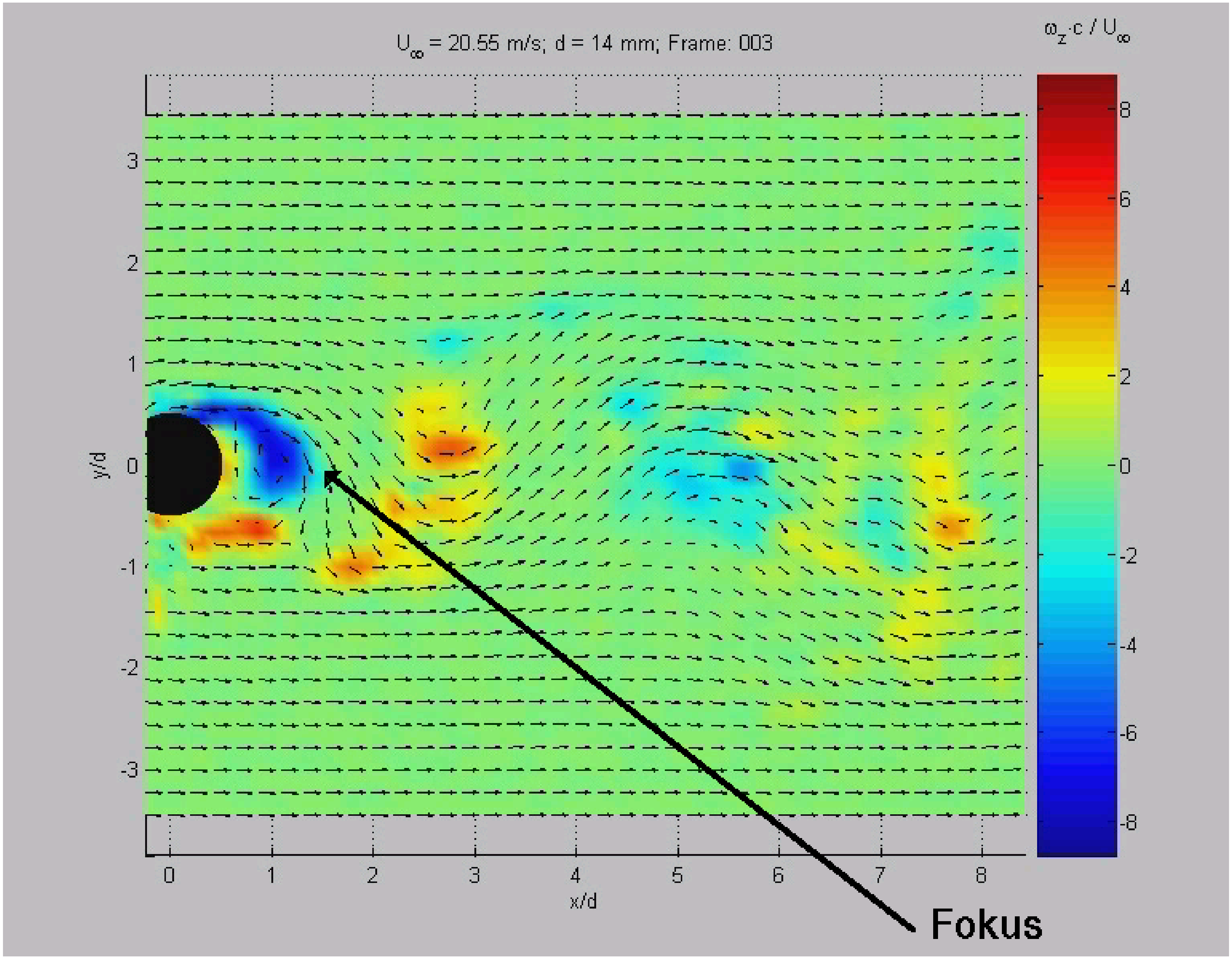
Instantaneous flow field



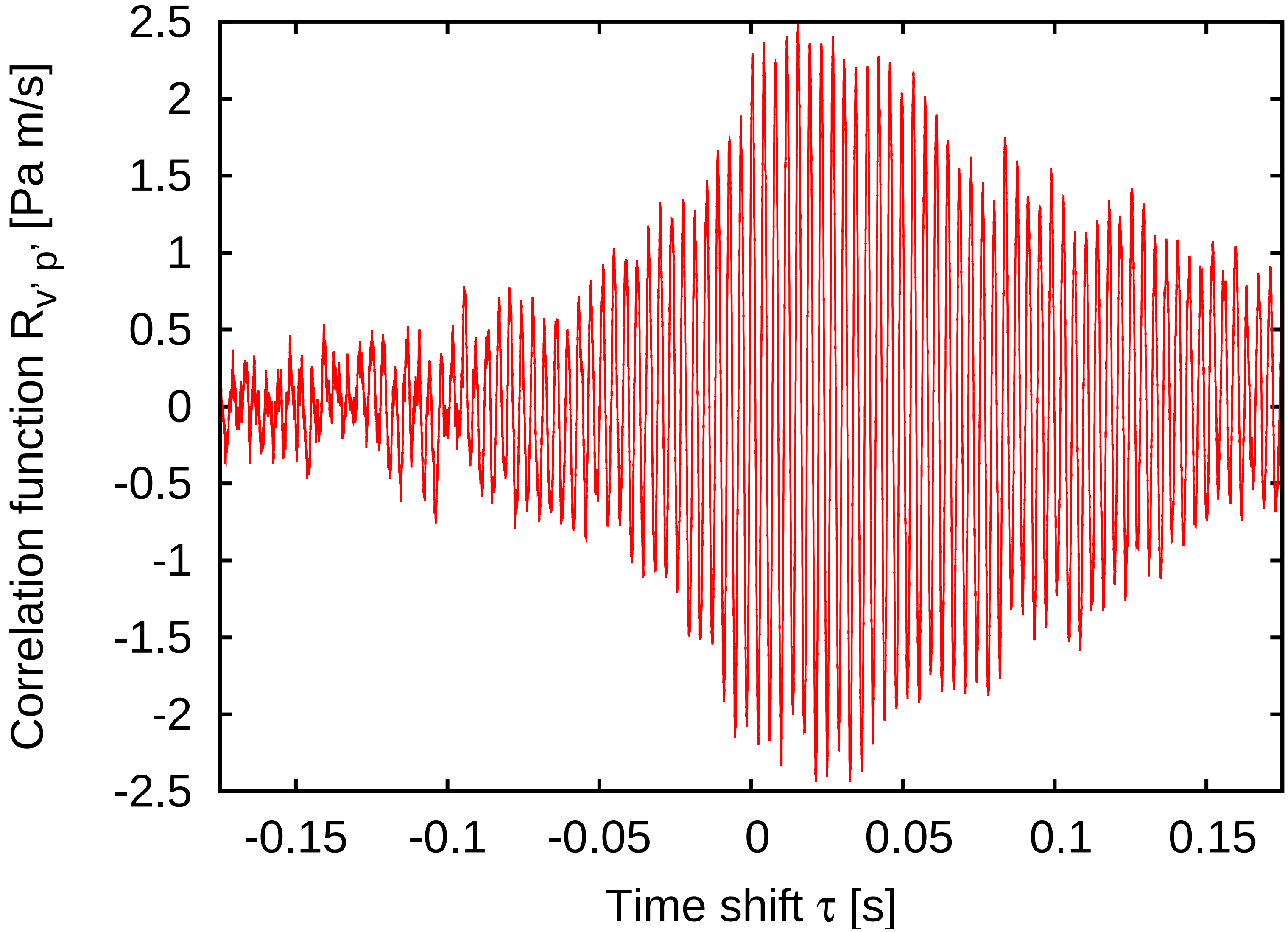
Sound-pressure spectrum



Correlation function for selected position



Correlation function for selected position (v)



Position: $(x/d,\ y/d) = (1.65,\ 0.0)$

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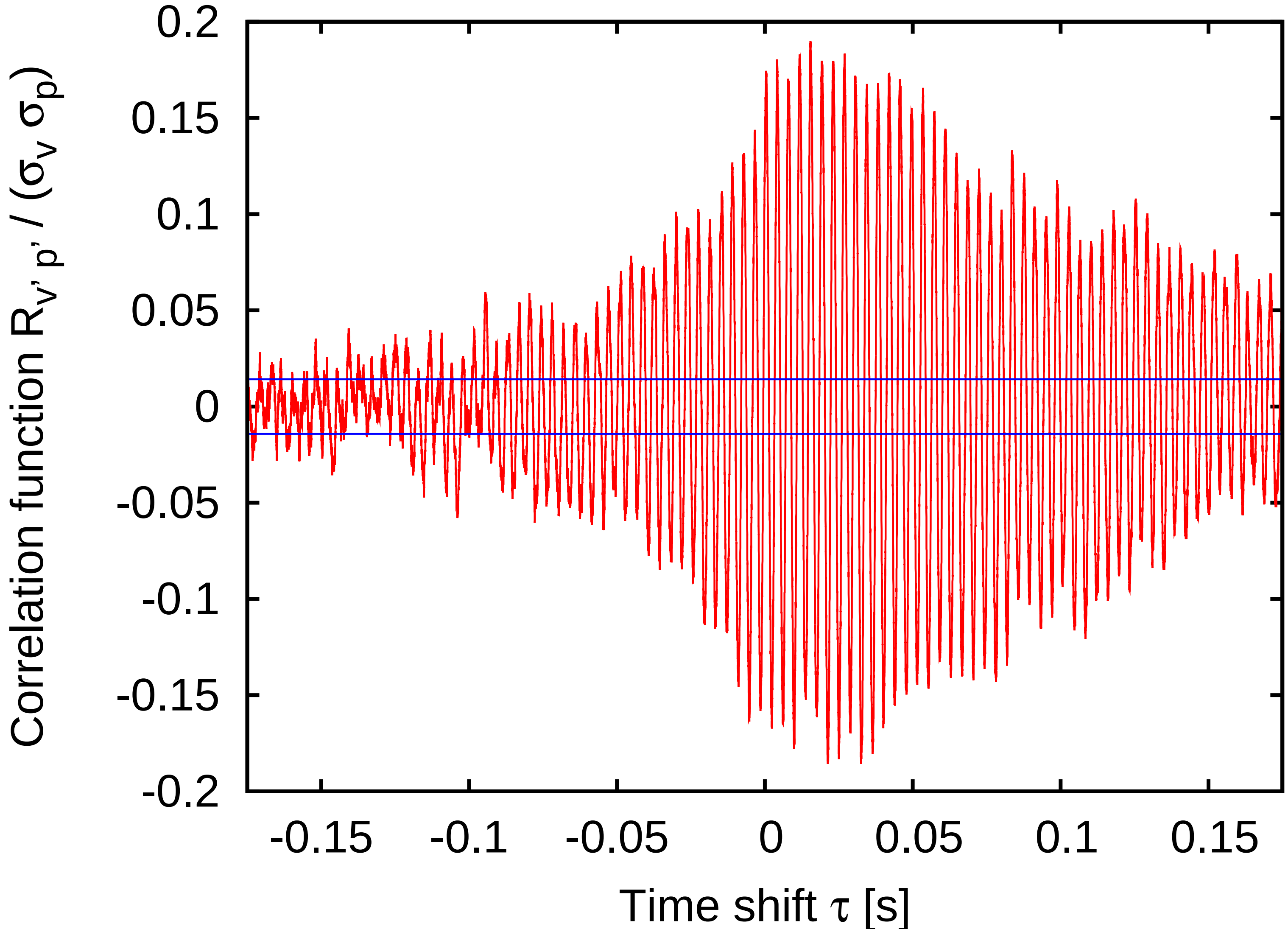
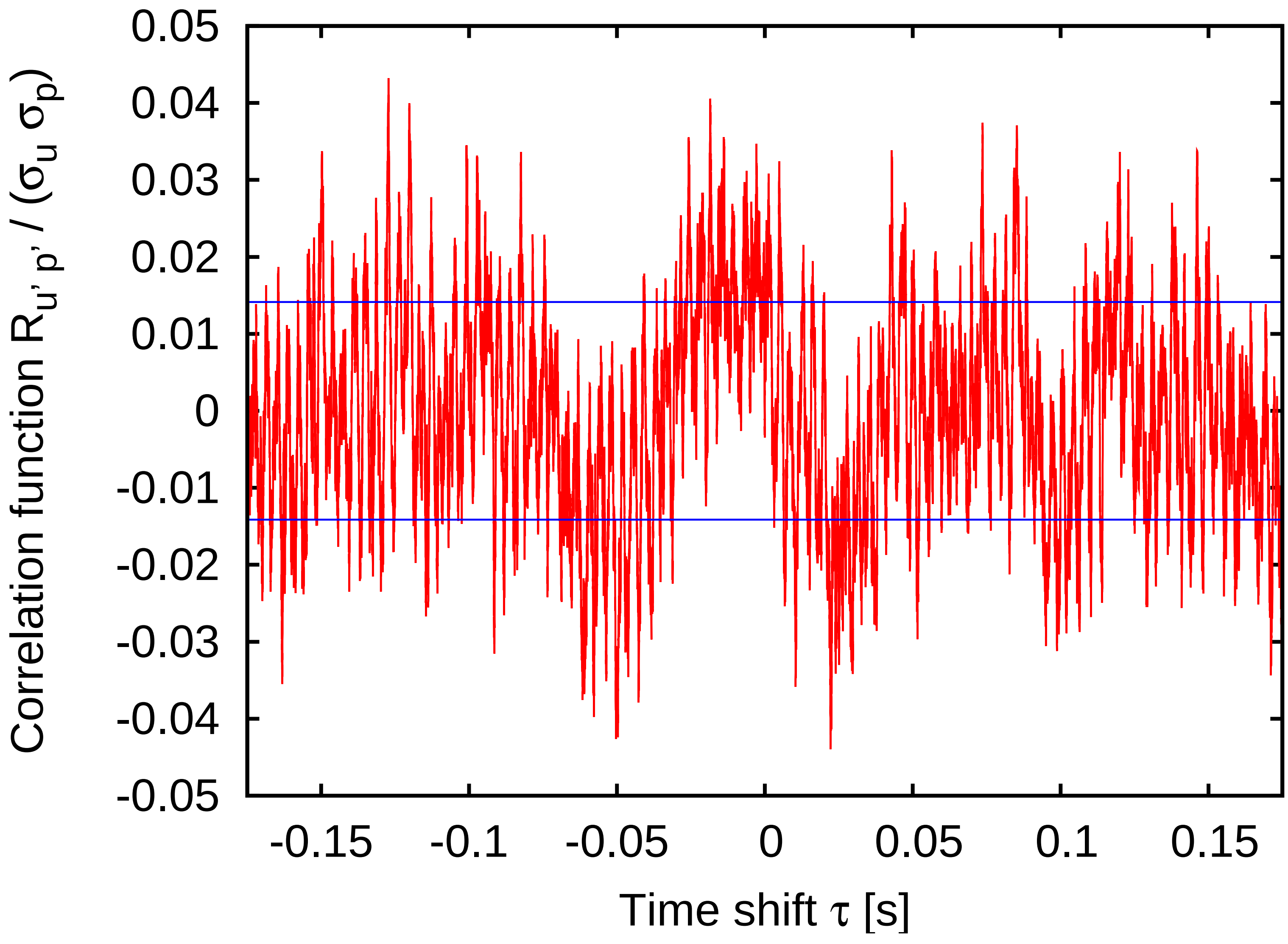
Correlation function for selected position (v)



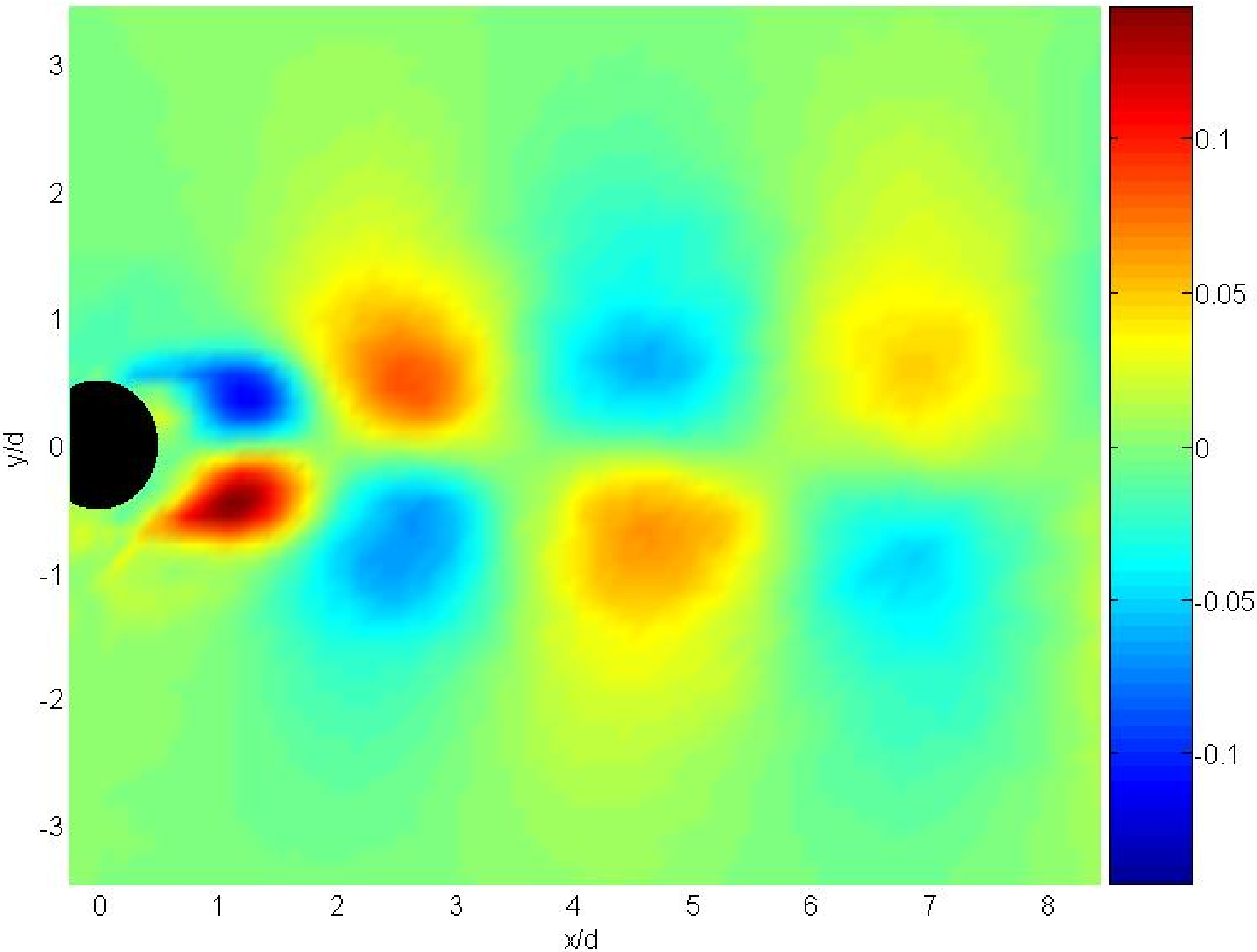
Normalization: $\sigma_v(\mathbf{x}) = \sqrt{\langle v'^2(\mathbf{x}, t)\rangle}$ and $\sigma_p = \sqrt{\langle p'^2(t)\rangle}$, variance $1/\sqrt{L}$

Correlation function for selected position (u)



Normalization: $\sigma_u(\mathbf{x}) = \sqrt{\langle u'^2(\mathbf{x},t) \rangle}$ and $\sigma_p = \sqrt{\langle p'^2(t) \rangle}$, variance $1/\sqrt{L}$

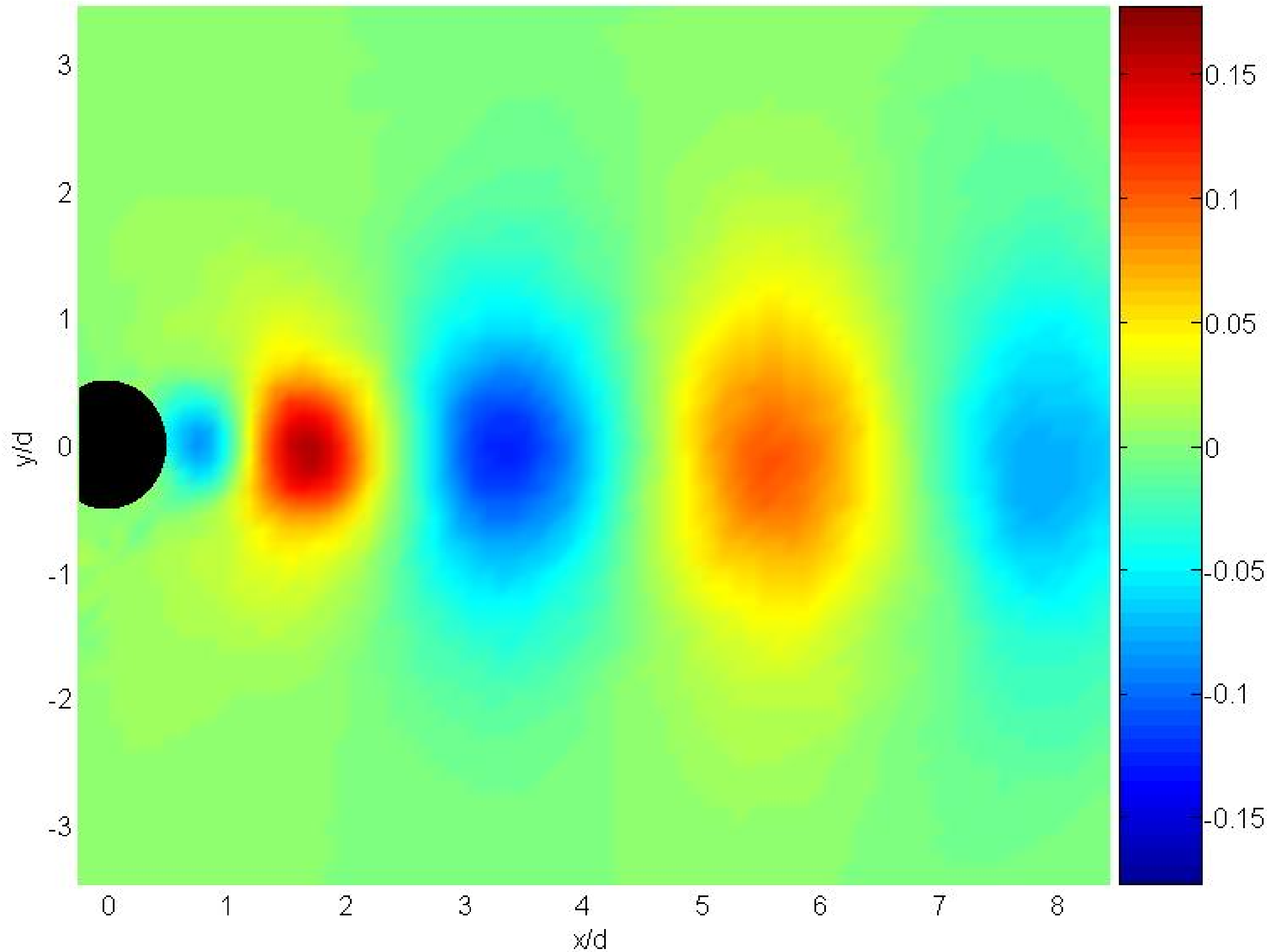
Spatial distribution of correlation (u)



$$\frac{R_{u'p'}}{\sigma_u^{(\max)}\sigma_p}$$

$R_{u'p'}(\mathbf{x}, \tau = 0)$ normalized using $\sigma_u^{(\max)} = \max\{\sigma_u(\mathbf{x})\}$ and σ_p

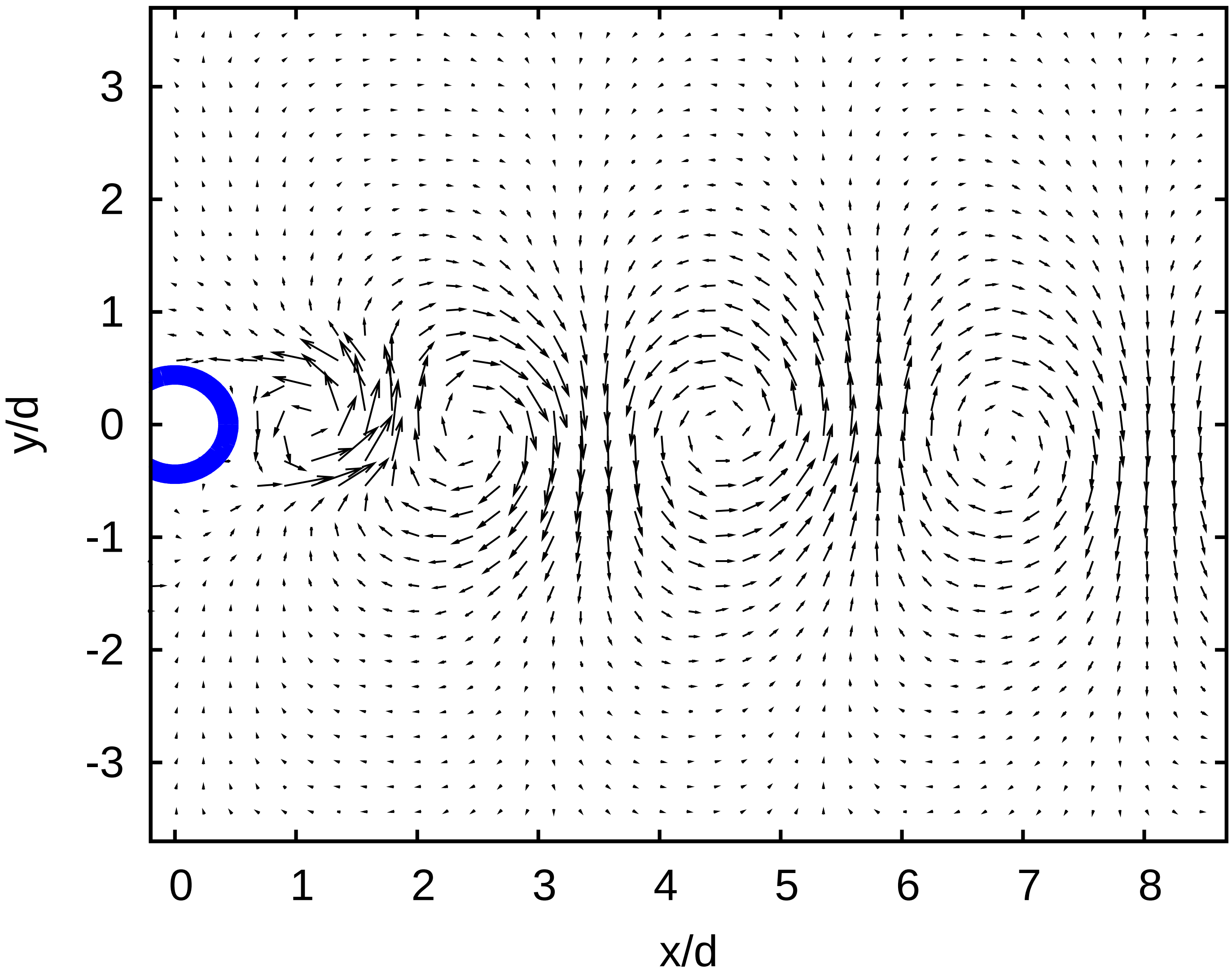
Spatial distribution of correlation (v)



$$\frac{R_{v'p'}}{\sigma_v^{(\max)}\sigma_p}$$

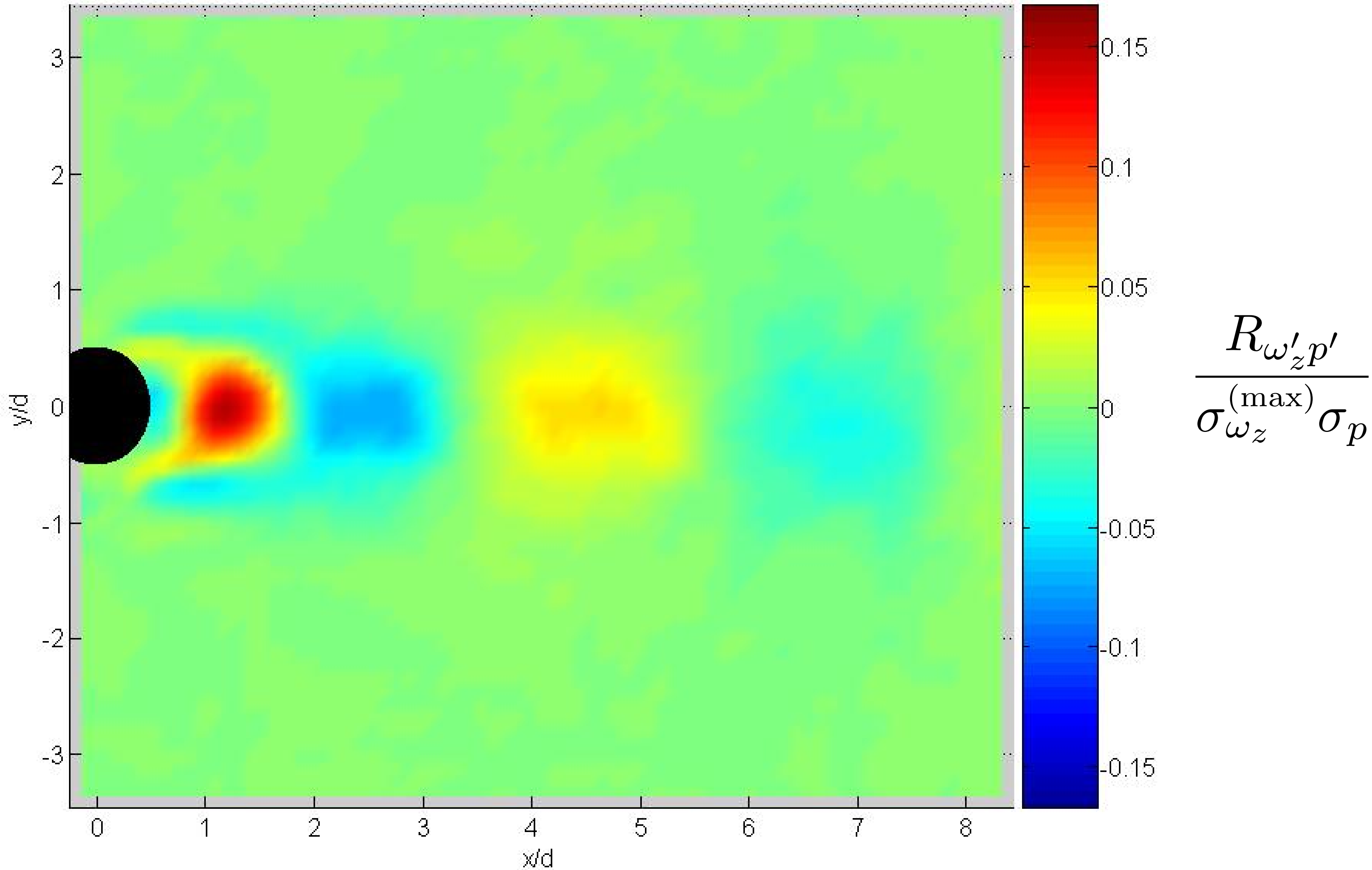
$R_{v'p'}(\mathbf{x}, \tau = 0)$ normalized using $\sigma_v^{(\max)} = \max\{\sigma_v(\mathbf{x})\}$ and σ_p

Spatial distribution of correlation (u,v)



Vectors: $(R_{u'p'}, R_{v'p'})$

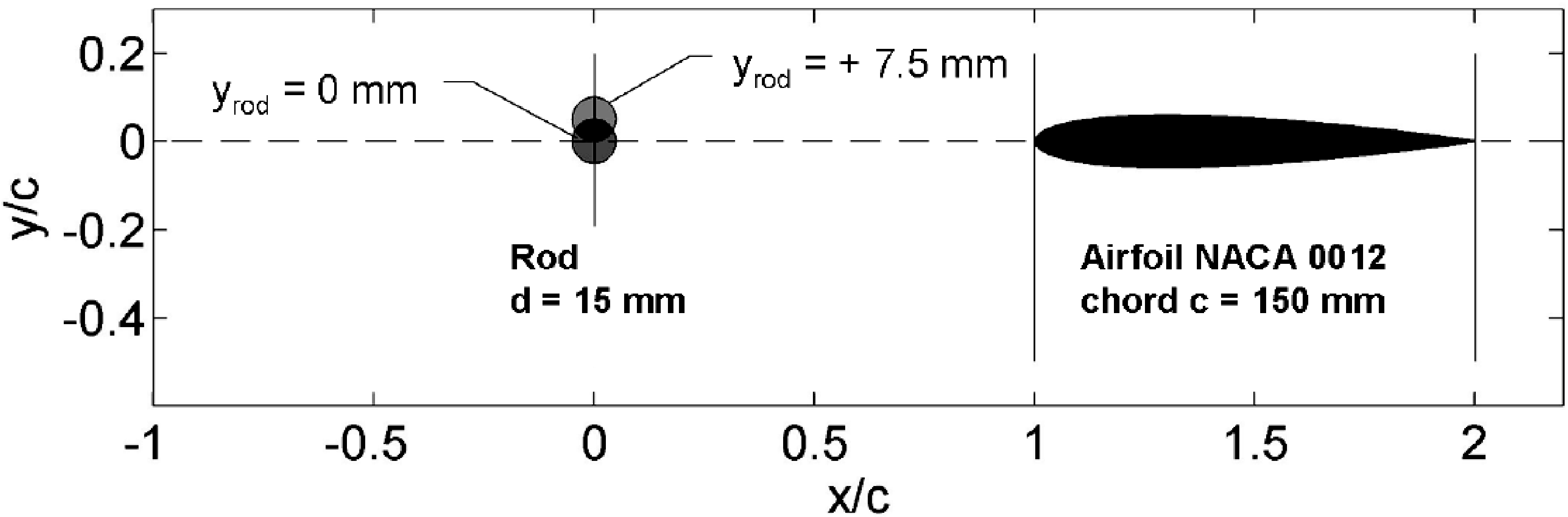
Spatial distribution of correlation (ω_z)



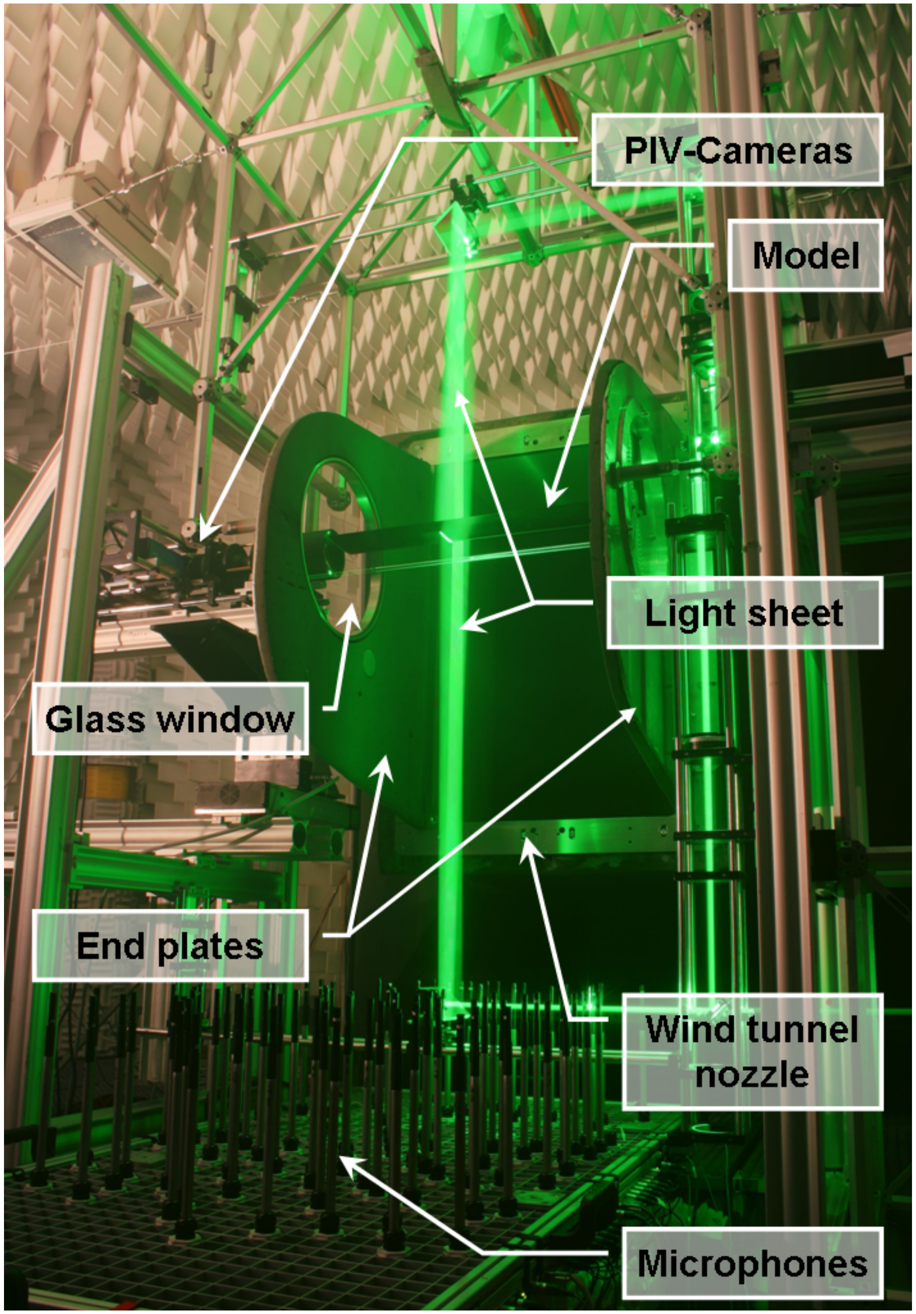
$R_{\omega'_z p'}(\mathbf{x}, \tau = 0)$ normalized using $\sigma_{\omega_z}^{(\max)} = \max\{\sigma_{\omega_z}(\mathbf{x})\}$ and σ_p

Second example: Rod-airfoil interaction

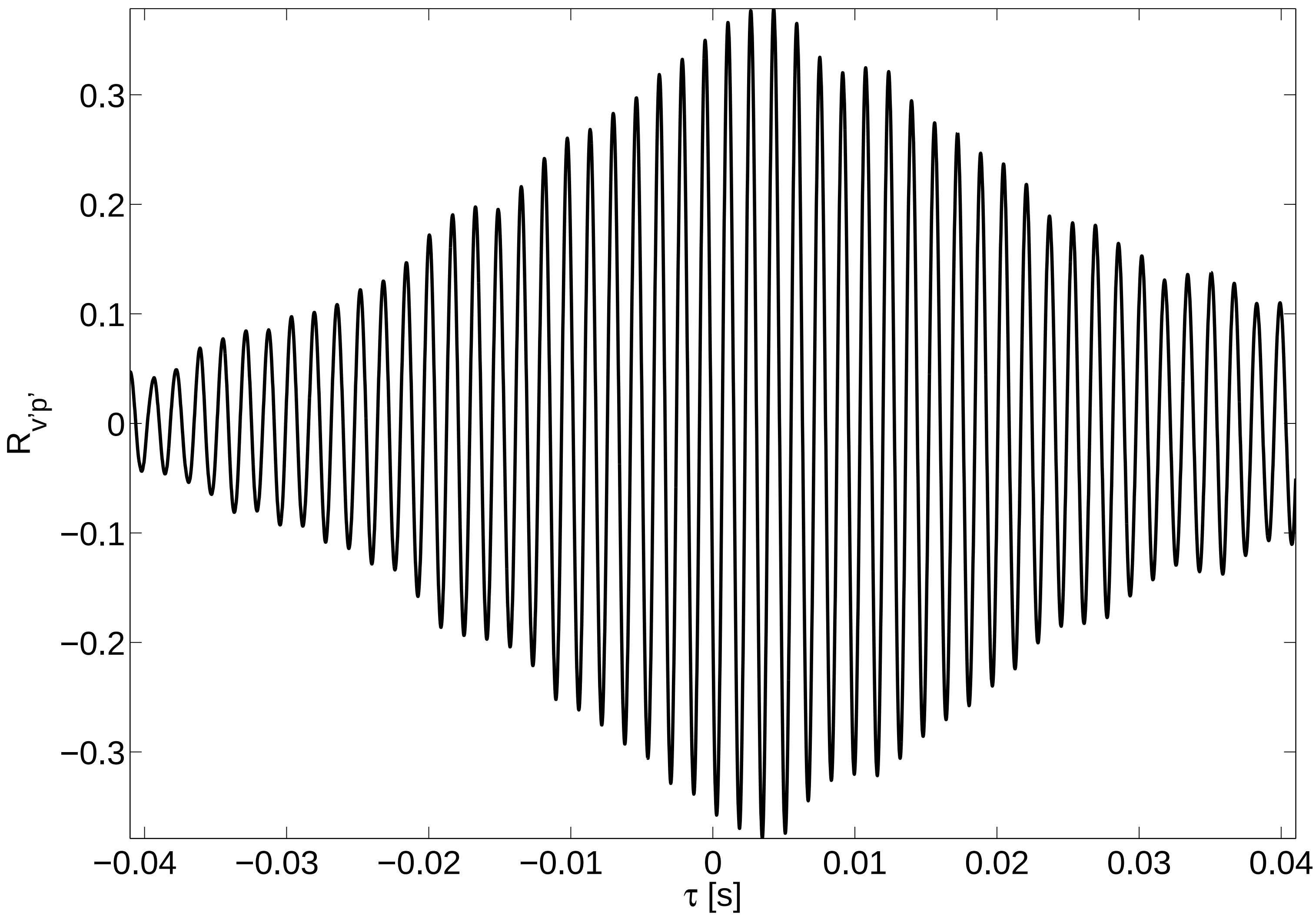
- Rod-airfoil (Henning et al., AIAA-Paper 2009-3184)
  - cylinder, diameter 15 mm
  - NACA 0012 airfoil, cord length 150mm
  - free-flow velocity 50 m/s
  - microphone array, 96 microphones, correlation with array output



Experimental setup

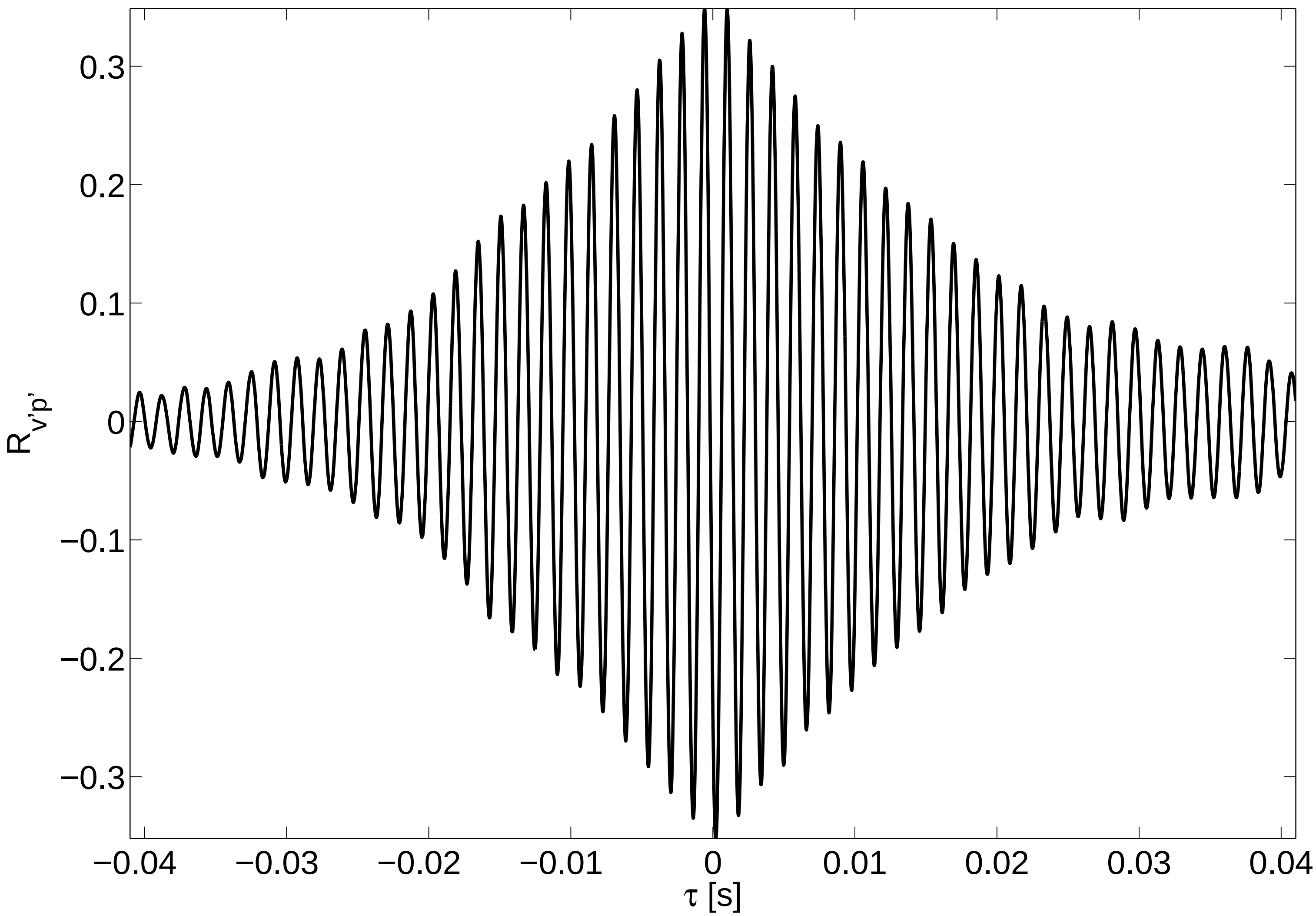


Correlation function (v) for position at cylinder



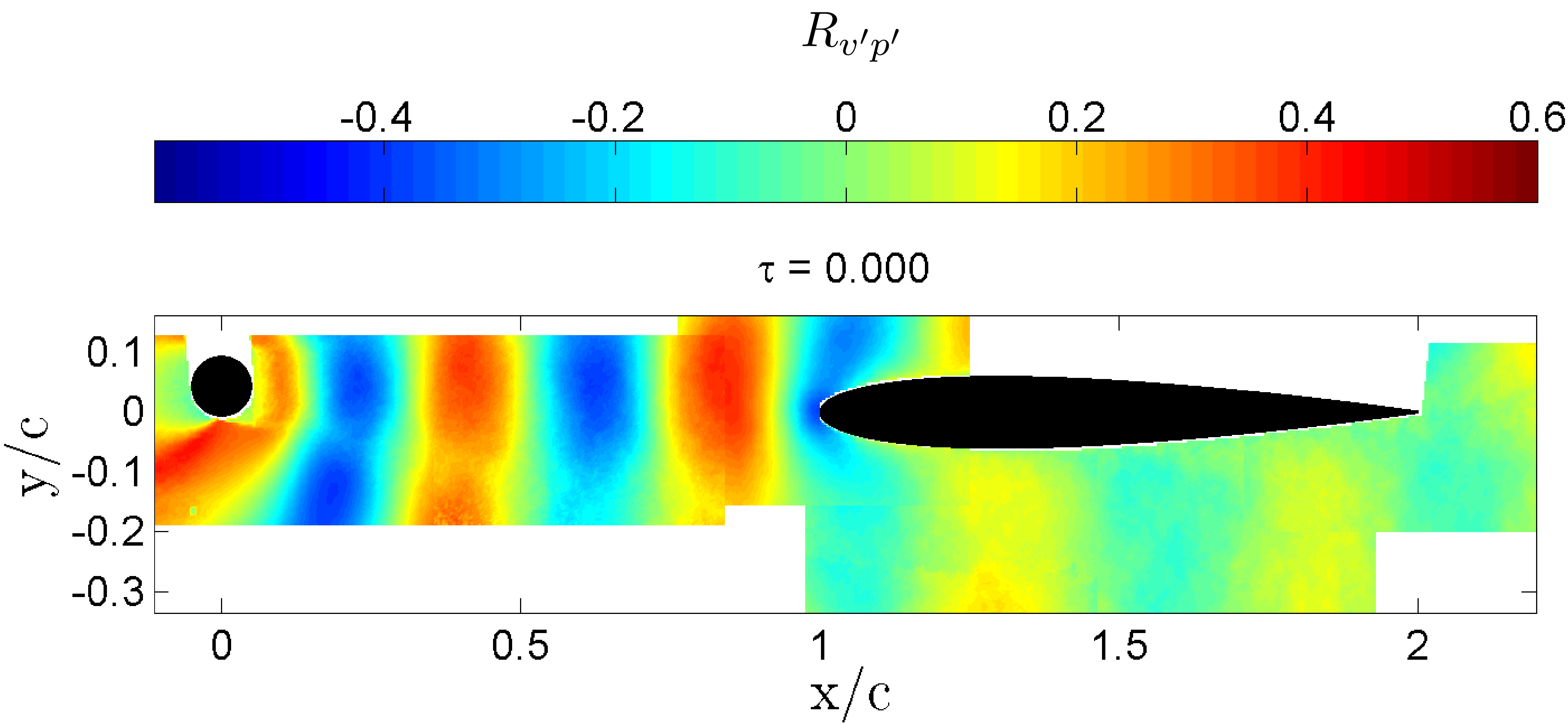
Position: $(x/c,\ y/c) = (0.165,\ 0.05)$

Correlation function (v) for position at airfoil

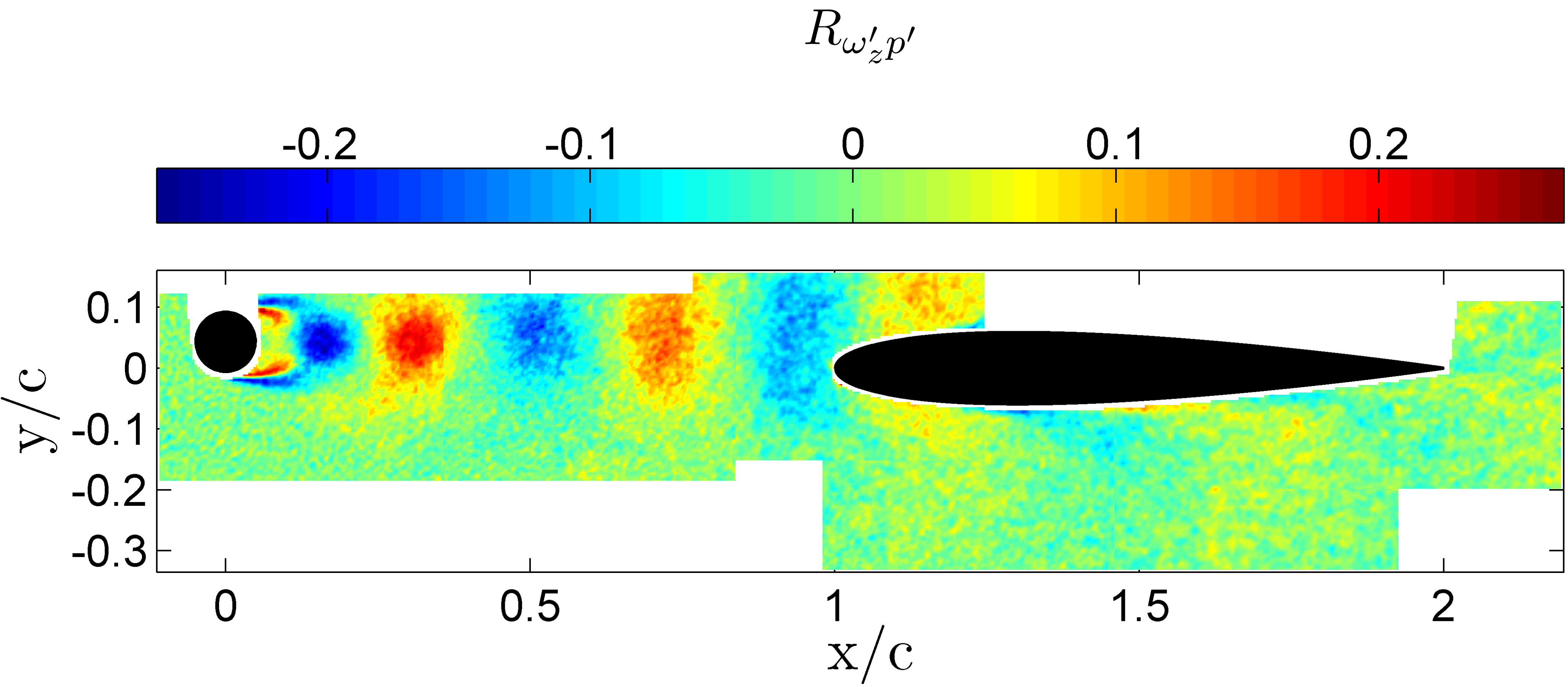


Position: $(x/c,\ y/c) = (0.95,\ 0.0)$

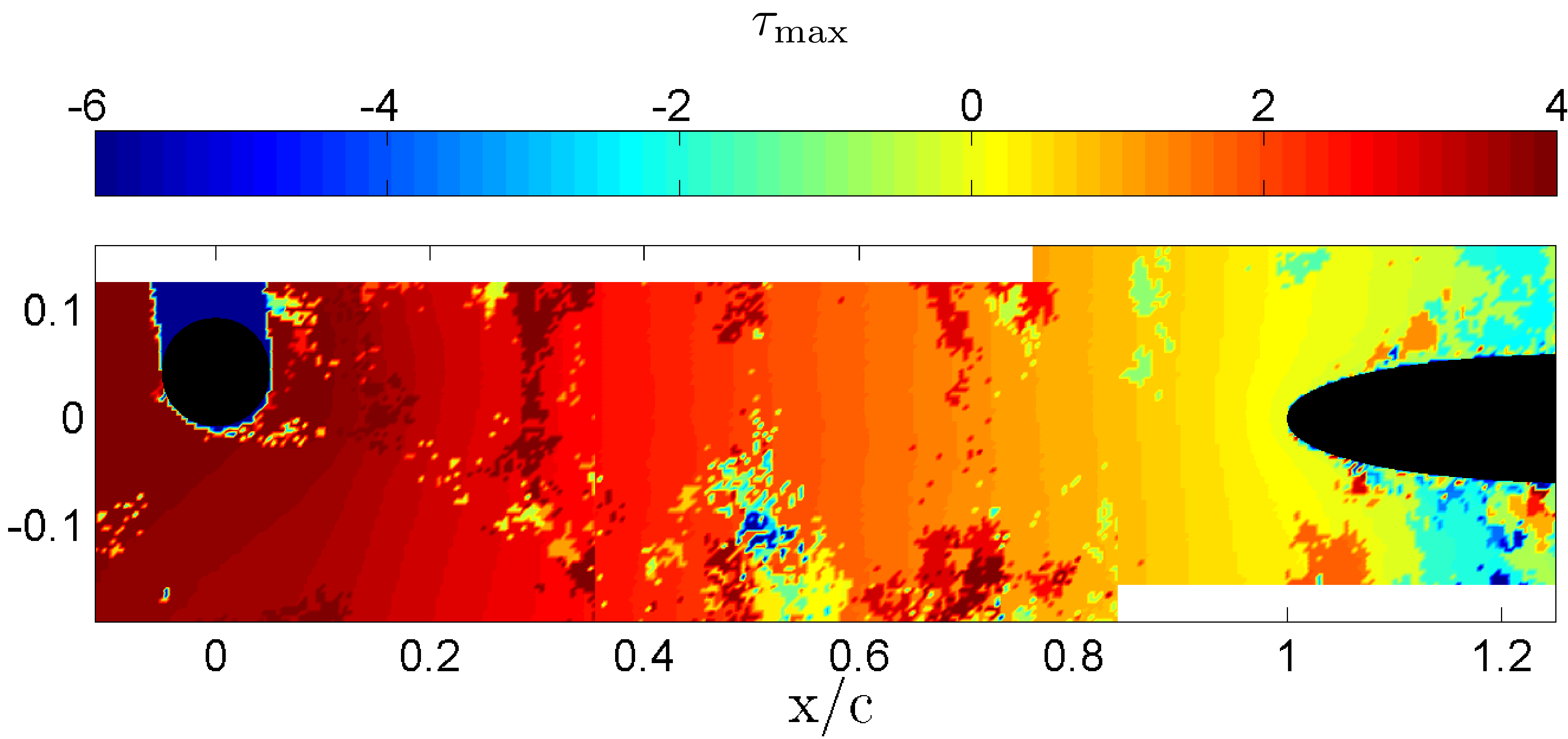
Spatial distribution of correlation (v)



Spatial distribution of correlation (ω_z)



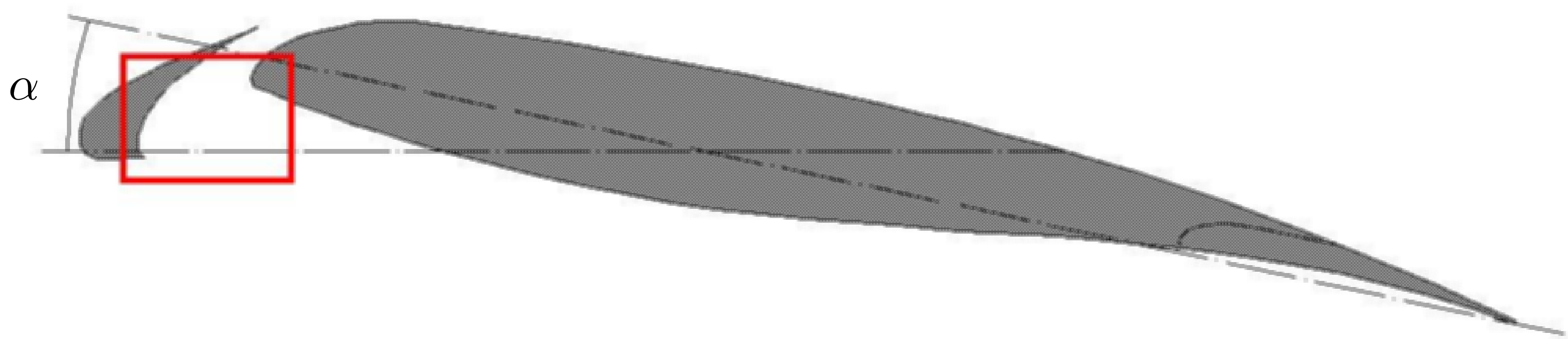
Spatial distribution of time delay ($\tau_{\max}$)



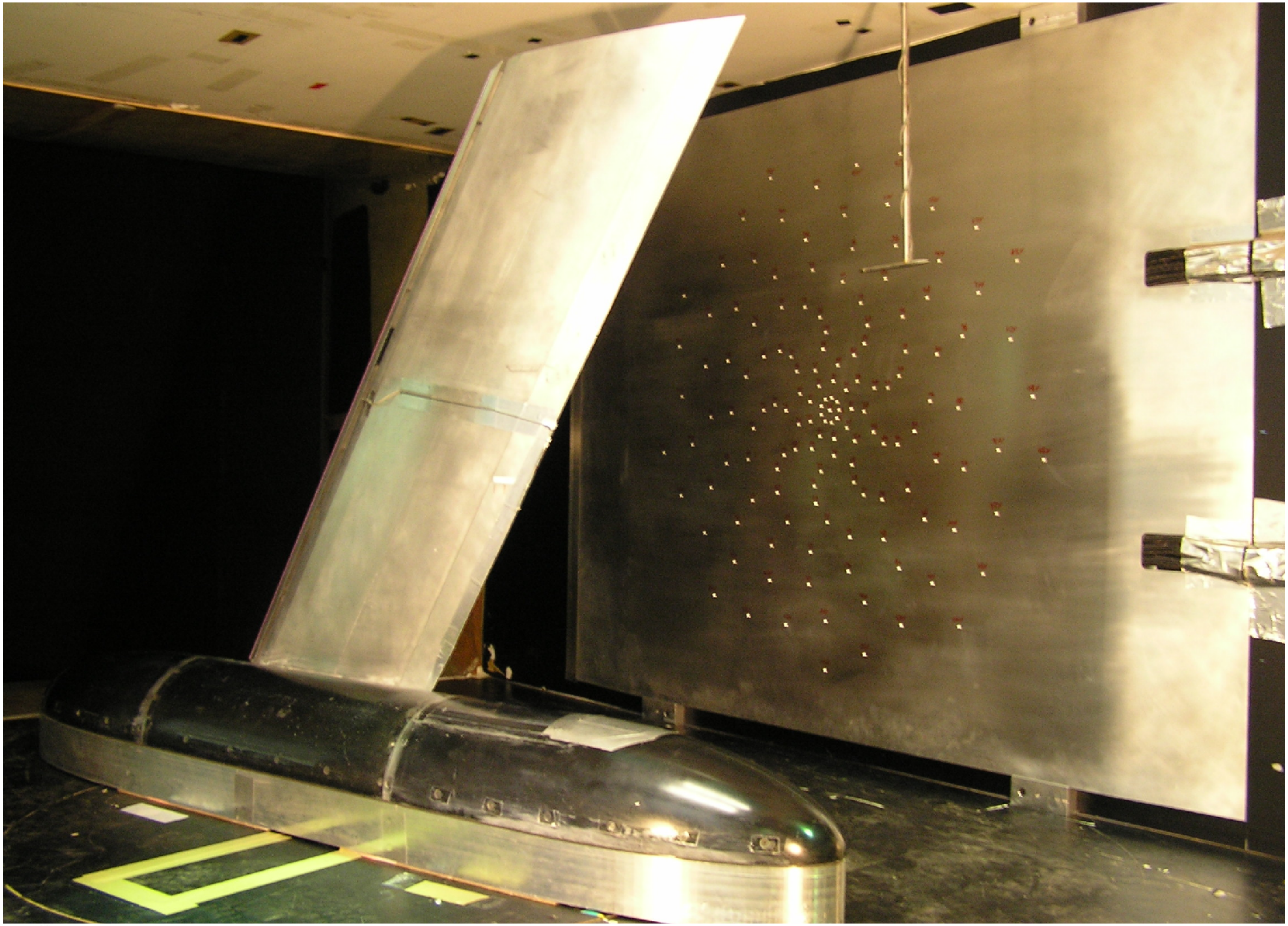
$\tau_{\max}$: Time delay at which $R_{v'p'}$ reaches maximum amplitude

Third example: Slat noise

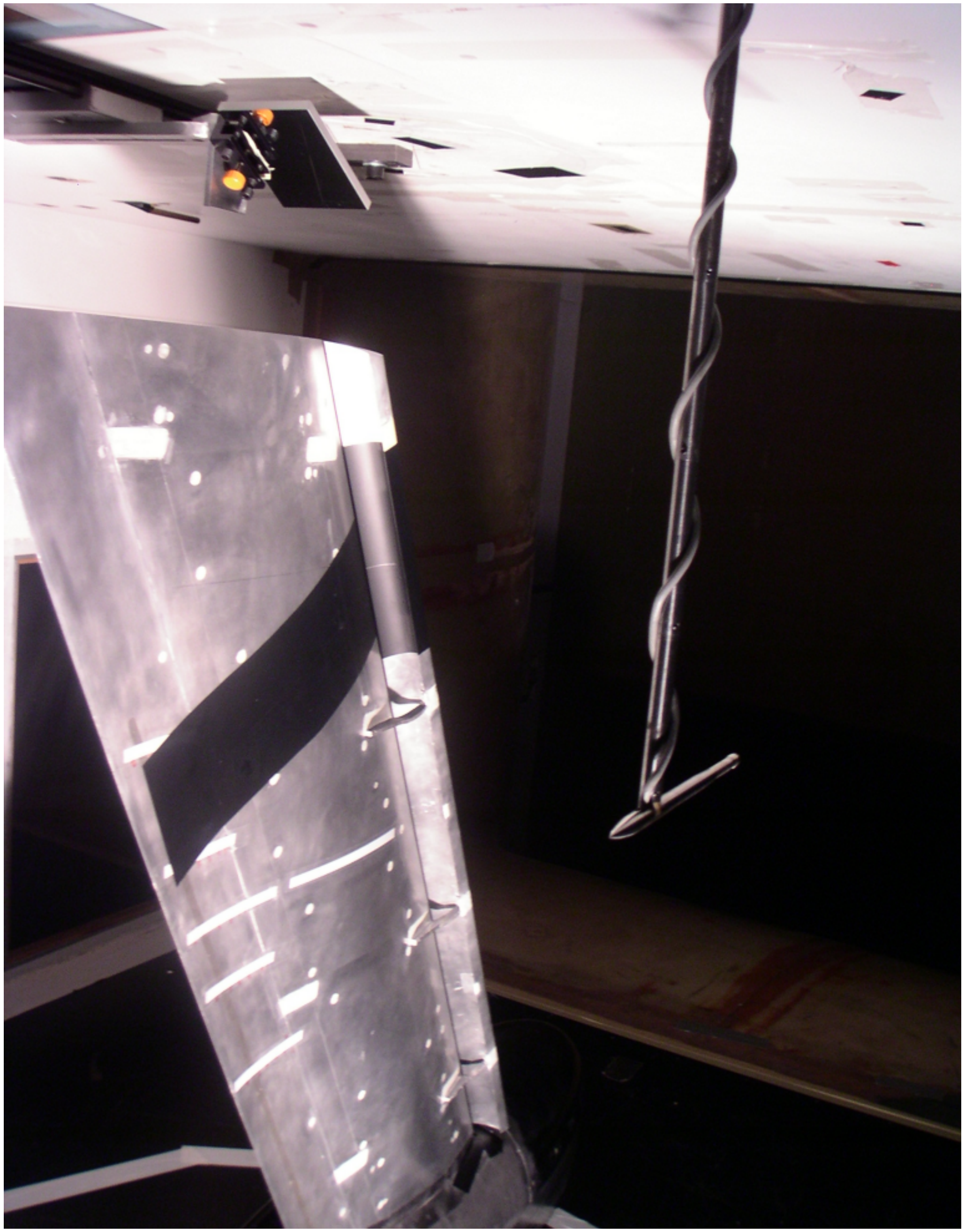
- Slat flow
  - "Swept Wing Constant Cord Half"-Modell (SCCH-Modell)
  - Light sheet perpendicular to leading edge
  - Angle of attack $\alpha = 16^\circ$
  - Free-flow velocity $U = 20\,\mathrm{m/s}$
  - Number of PIV snapshots $L = 5000$



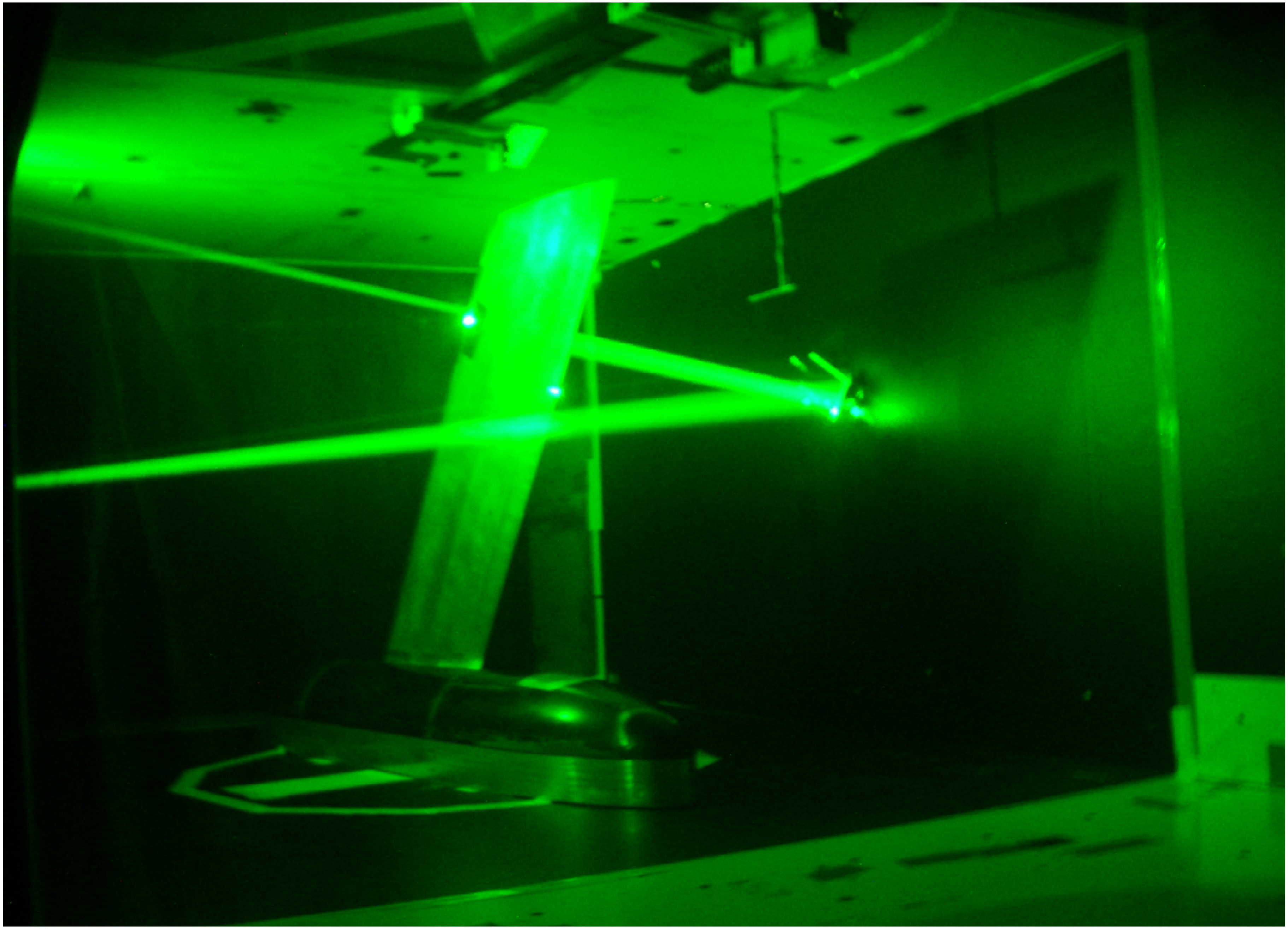
Experimental setup



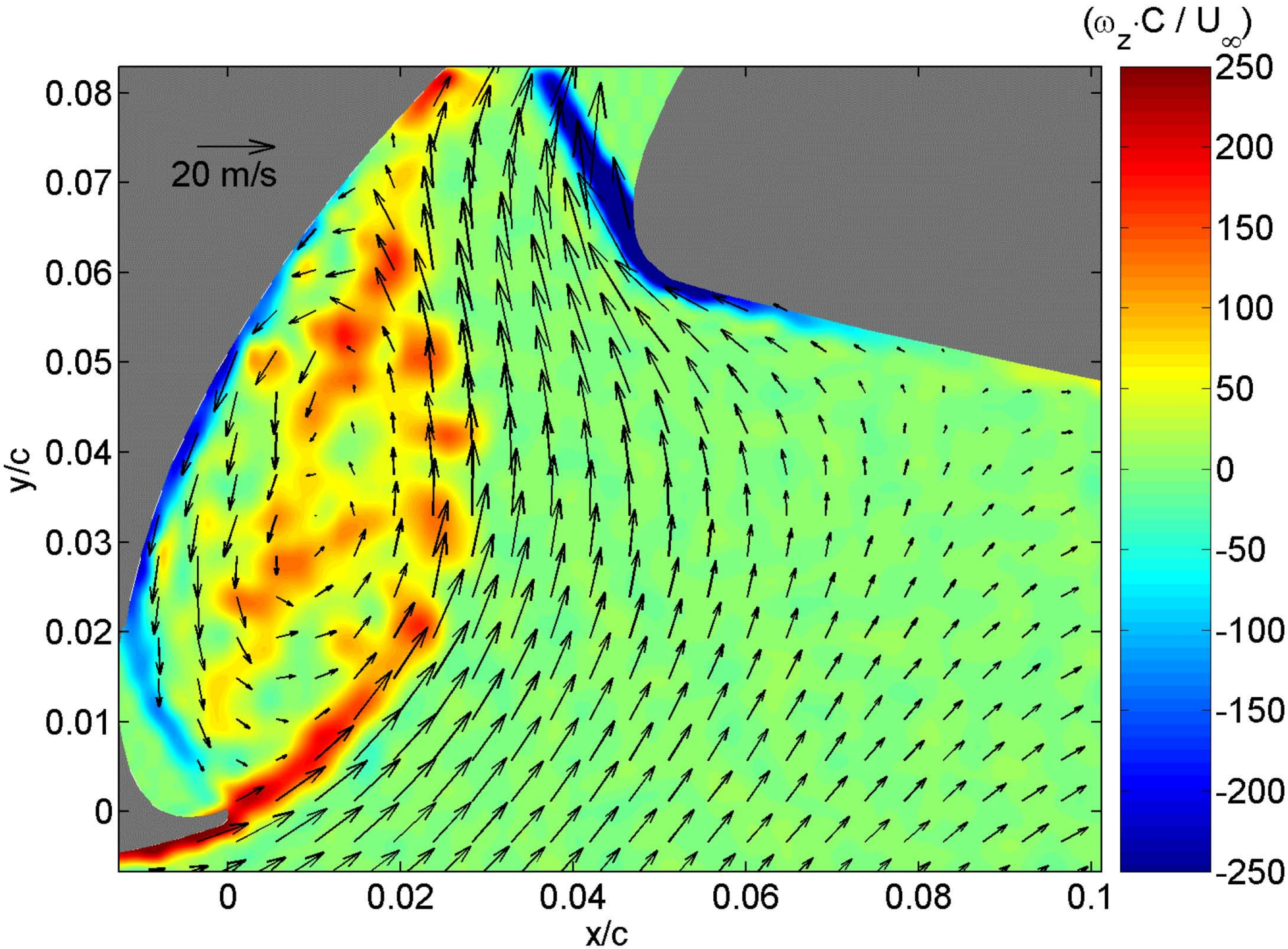
Experimental setup



Experimental setup



Instantaneous flow field



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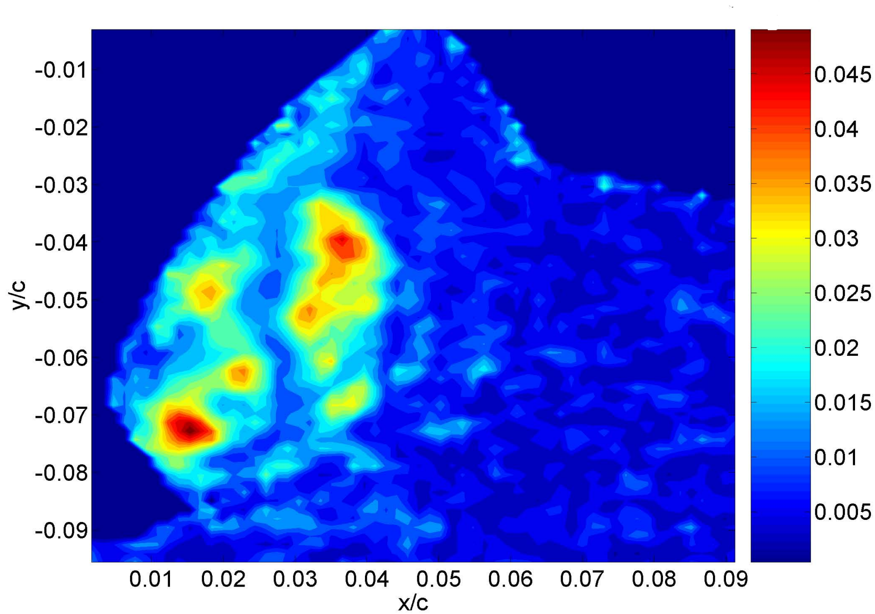
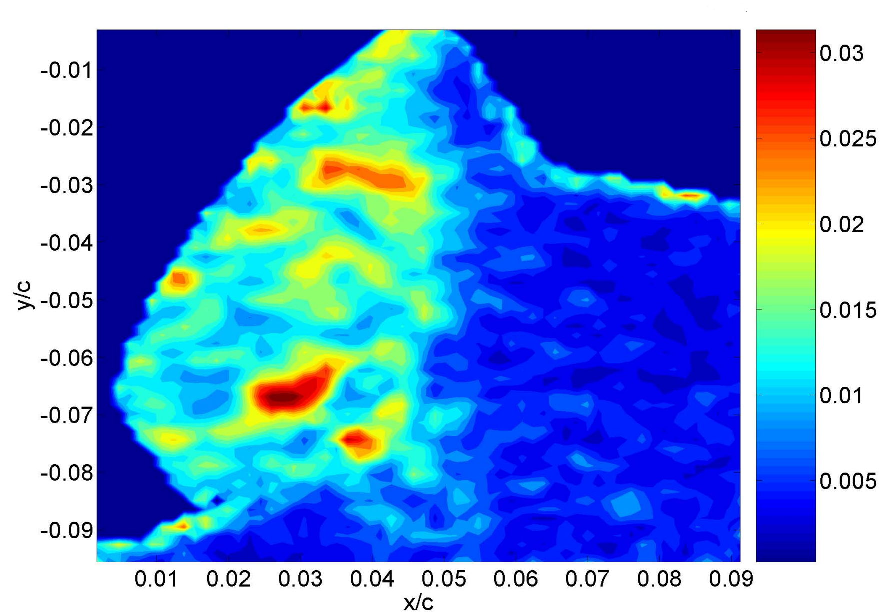
Spatial distribution of the correlation

- Mean strength of correlation function
  - Averaging over interval -0.01 s $\leq \tau \leq$ 0.01 s

$$\frac{\left\langle R^2_{u'p'}(\mathbf{x},\tau)\right\rangle_\tau^{-1/2}}{\sigma_u(\mathbf{x})\,\sigma_p} \qquad \frac{\left\langle R^2_{v'p'}(\mathbf{x},\tau)\right\rangle_\tau^{-1/2}}{\sigma_v(\mathbf{x})\,\sigma_p}$$





Local source efficiency

- Correlation results give idea about source process
- But: No local measure of source efficiency
- Desirable: Local quantitative measure for source efficiency
  - How much contribute sources at a certain position to the far field
  - Source strength without reactive power
  - Average value
- Questions:
  - Is it possible to find a reasonable definition for a local measure of source efficiency ?
  - Is it possible to obtain such a value from PIV experiments ?

Fluctuation intensity

Model: Lighthills acoustic analogy

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2\,\Delta\right)\rho' = \frac{\partial T_{ij}}{\partial x_i \partial x_j}$$

Inhomogeneous wave equation:

$$\frac{\partial^2 \phi}{\partial t^2} - c_0^2 \Delta \phi = q$$

Energy conservation:

$$\frac{\partial e}{\partial t} + \mathrm{div}\,\vec{I} = \frac{\partial \phi}{\partial t}\, q \equiv q_e$$

with

$$e = \frac{1}{2}(\dot{\phi})^2 + \frac{c_0^2}{2}(\mathrm{grad}\phi)^2 \qquad \text{and} \qquad \vec{I} = -c_0^2\,\dot{\phi}\,\mathrm{grad}\phi$$

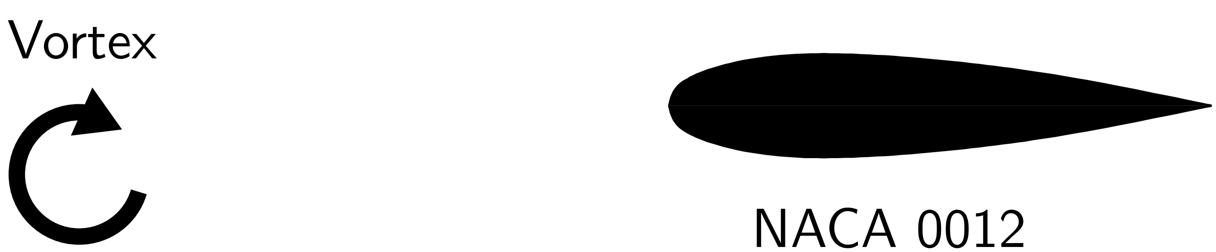
Local source efficiency

Average production rate of fluctuation energy

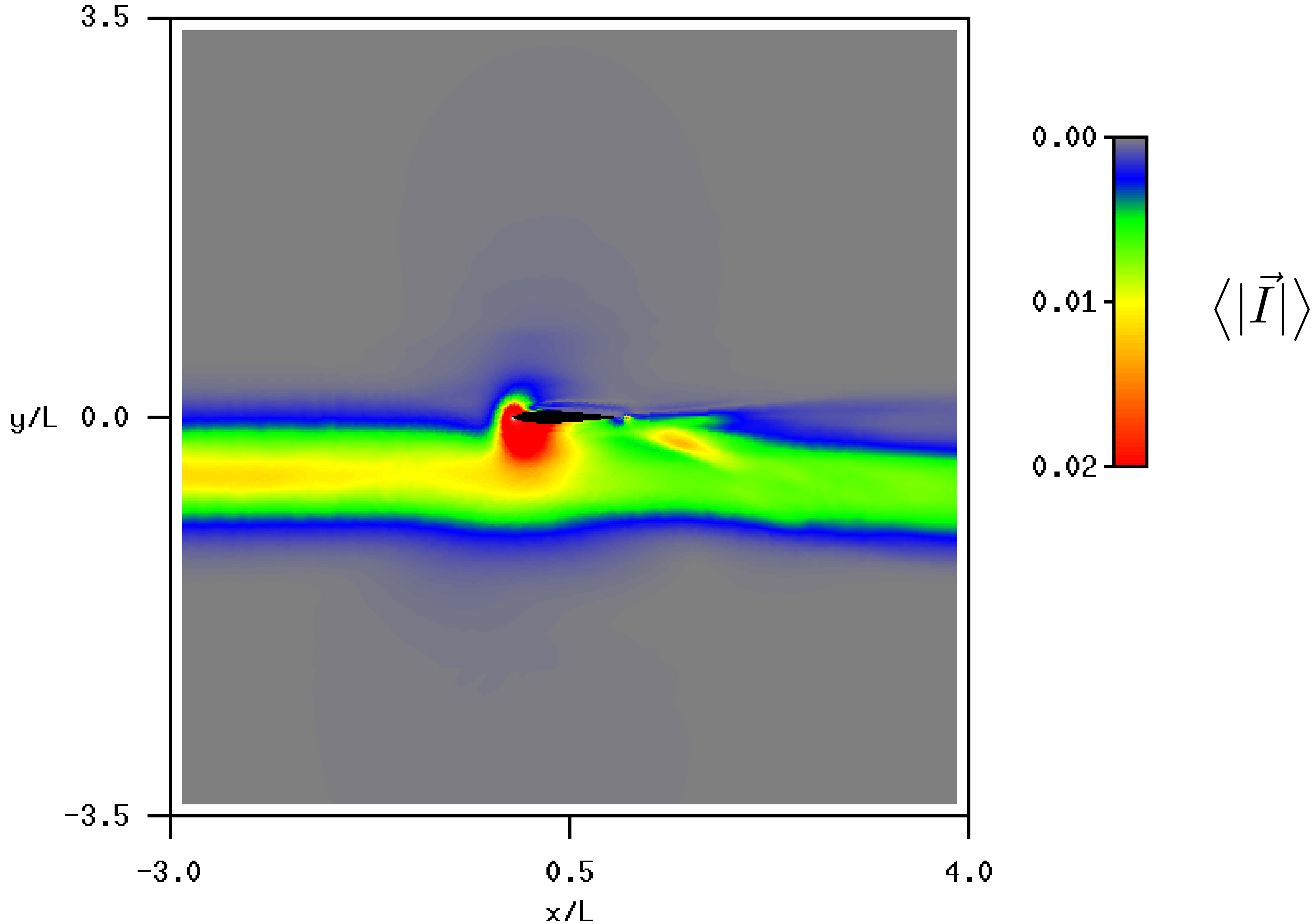
$$\langle \operatorname{div} \vec{I} \rangle = \langle q_e \rangle$$

Angle brackets: Averaging over certain time

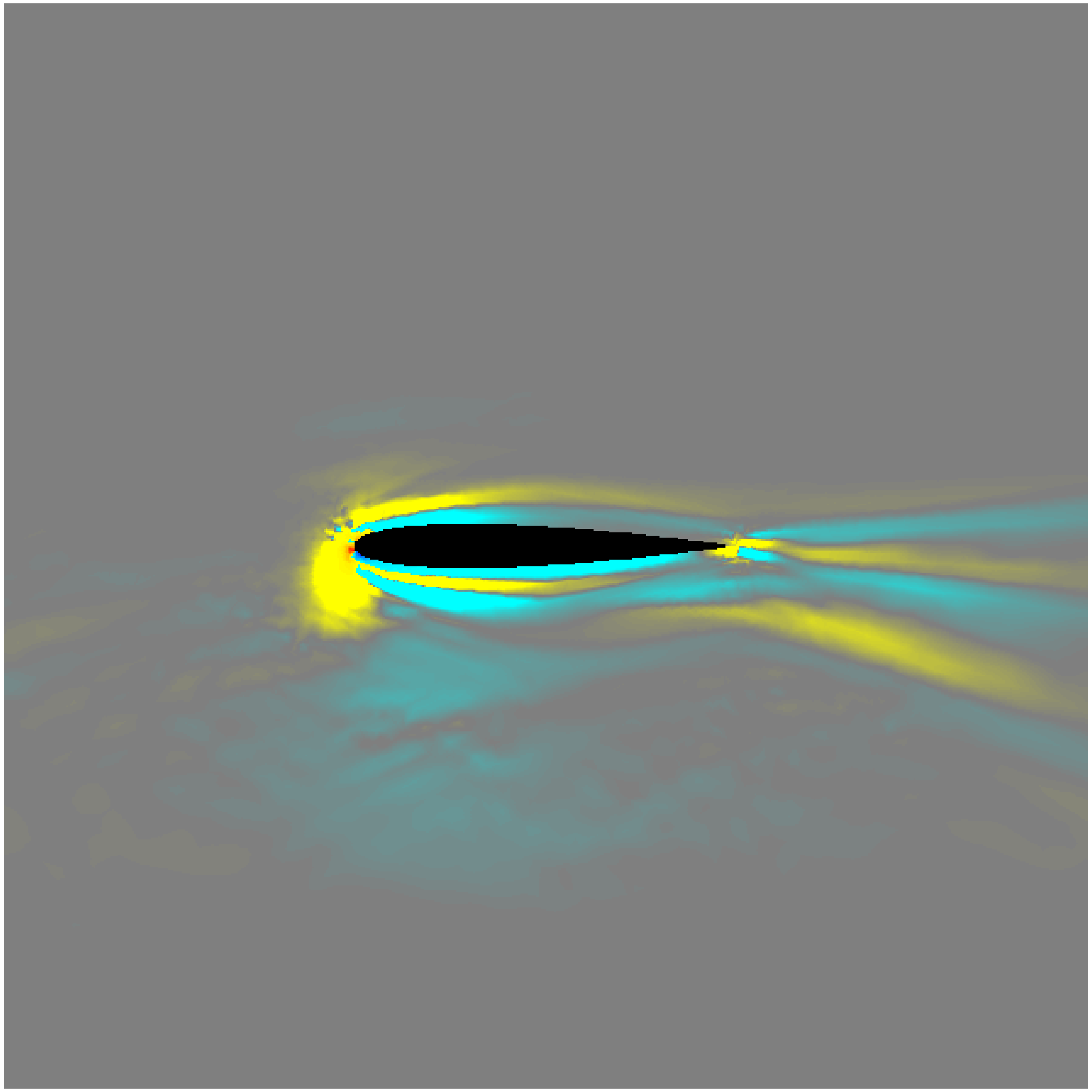
- Local value
- Depends on model equation
- Numerical example: Vortex-airfoil interaction (M $=$ 0.3)



Average magnitude of intensity



Local source efficiency



$$\left\langle \mathrm{div}\vec{I} \right\rangle$$

Alternative acoustic variable

Inhomogeneous wave equation:

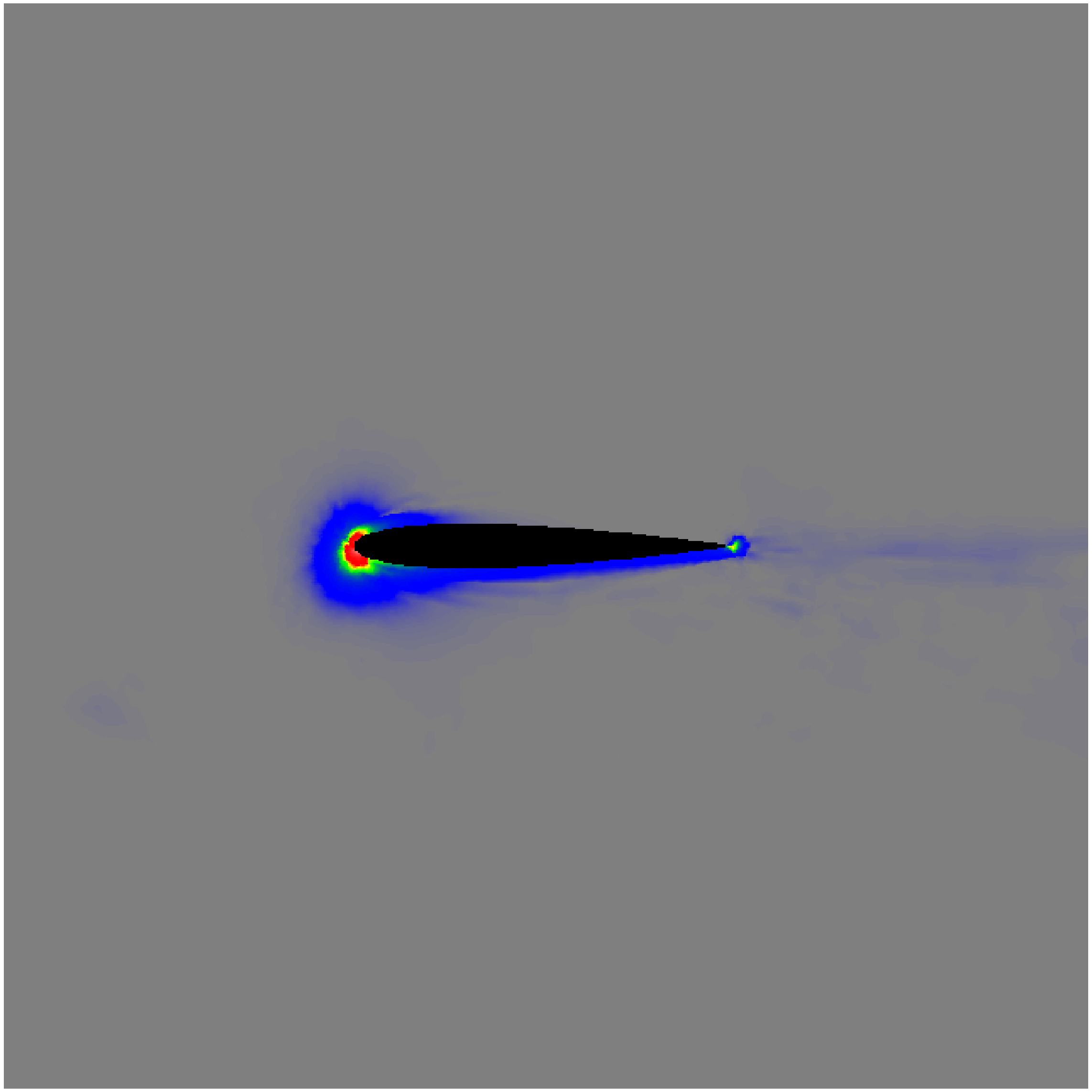
$$\frac{\partial^2 \phi}{\partial t^2} - c_0^2 \Delta \phi = q$$

Acoustic valiable

$$\phi = \mathsf{div}\, \vec{v}$$

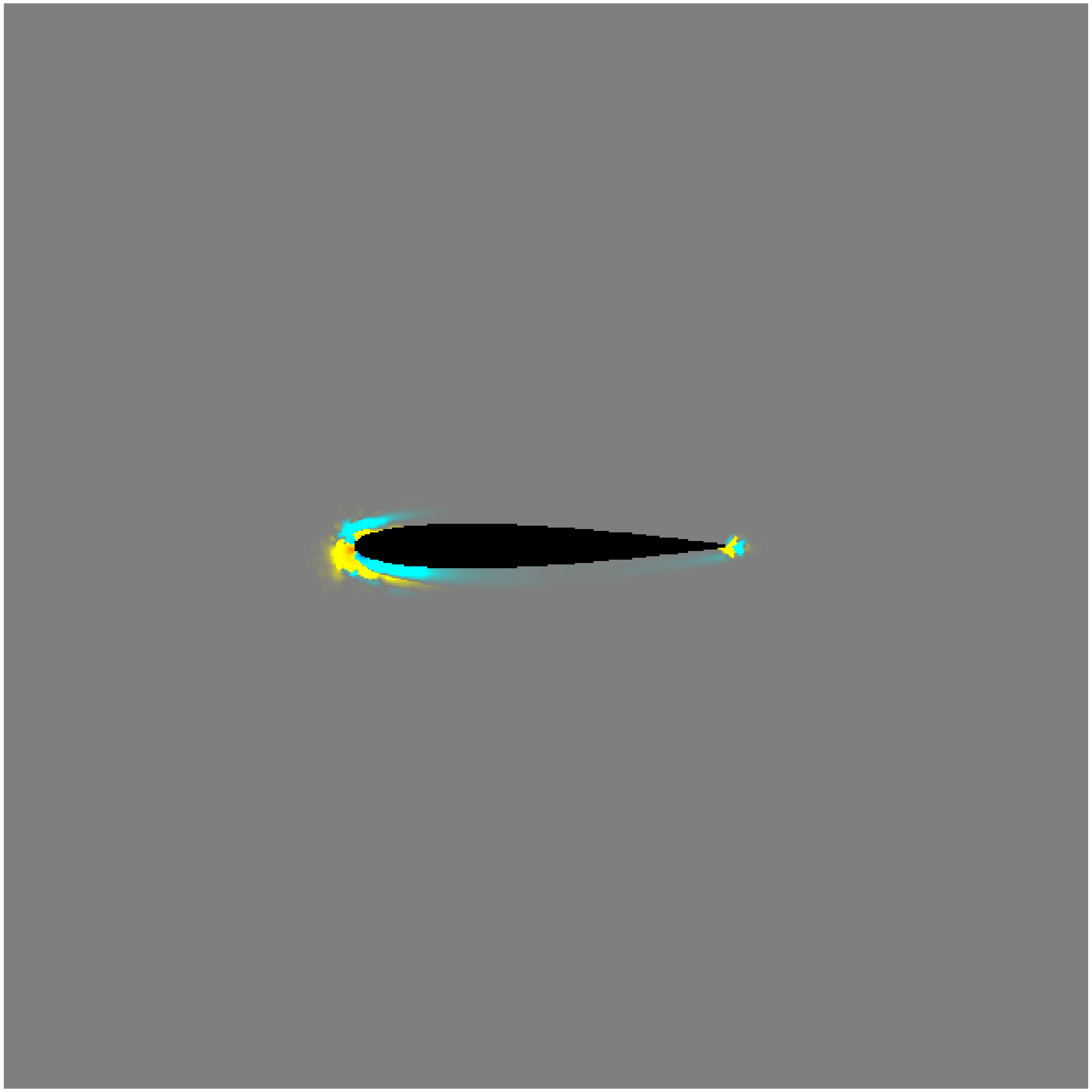
- Little div $\vec{v}$ in a convected vortex
- In the far field variable div $\vec{v} = \dot{\rho}/\rho + \mathcal{O}(2)$
- Theoretically div $\vec{v}$ can be obtained from PIV measurements
- In practice $R_{u',p'}, R_{v',p'}$ instead of $\vec{v}$

Average magnitude of intensity



$$\langle |\vec{I}| \rangle$$

Local source efficiency



$$\left\langle \mathrm{div}\vec{I} \right\rangle$$

Summary

- Synchronized measurement of velocity and far field pressure using PIV and microphone is feasible
- Calculation of correlation between near and far field possible
- Cylinder
  - 5000 PIV snapshots sufficient
  - Flow structures in source region identified
- Slat flow
  - 5000 PIV snapshots not sufficient
- Improvements
  - 3D-tomographic PIV
  - Fluctuation intensity

Acknowledgment

- Contributors / Co-workers
  - Arne Henning
  - Kristian Käpernick
  - Lars Koop
  - Stefan Kröber
  - Andreas Lauterbach

Fluctuation intensity

Lighthills acoustic analogy:

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Theoretische Grundlagen

Lighthill's akustische Analogie

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2\,\Delta\right)\rho' = \frac{\partial T_{ij}}{\partial x_i \partial x_j}$$

Lighthill-Spannungen

$$T_{ij} = \rho v_i v_j + \tau_{ij} - \delta_{ij}\left\{p - c_0^2 \rho'\right\}$$

- Oft Approximation $T_{ij} \approx \rho_0 v_i v_j$ möglich
  - Keine großen Temperatur-Unterschiede zwischen Quellbereich und Fernfeld
  - Abweichungen von adiabatischen Schwankungen sind vernachlässigbar
  - Hohe Reynolds-Zahl, kleine Machzahl

Theoretische Grundlagen

Approximative Lösung nach Proudman (1952)

$$\rho'(\mathbf{x},t) = \frac{\rho_0}{4\pi c_0^4\,|\mathbf{x}|} \int\limits_V \frac{\partial^2 v_r^2}{\partial t^2}\,(\mathbf{y}, t-\tau_0)\,\mathrm{d}^3\mathbf{y}$$

v_r: Komponente der Geschwindigkeit vom Quellpunkt $\mathbf{y}$ in Richtung des Beobachters $\mathbf{x}$

V: Quellbereich

τ_0: Laufzeit, $\tau_0 = |\mathbf{x} - \mathbf{y}|/c_0$

- Voraussetzungen
  - Approximation $T_{ij} \approx \rho_0 v_i v_j$
  - Geometrisches Fernfeld: $|\mathbf{x}| \gg$ Ausdehnung von V
  - Keine Berandung, Koordinatenursprung im Quellgebiet

Theoretische Grundlagen

Korrelationsfunktion nach Lee und Ribner (1972)

$$R_{p'p'}(\mathbf{x},\tau) = \frac{\rho_0}{4\pi c_0^2\,|\mathbf{x}|} \int\limits_V \frac{\partial^2 R_{v_r^2 p'}}{\partial \tau^2}(\mathbf{x},\mathbf{y},\tau+\tau_0)\,\mathrm{d}^3\mathbf{y}$$

mit Autokorrelationsfunktion des Drucks am Beobachter

$$R_{p'p'}(\mathbf{x},\tau) = \langle p'(\mathbf{x},t)\,p'(\mathbf{x},t+\tau)\rangle$$

und Kreuzkorrelationsfunktion zwischen Nahfeldgröße v_r^2 und Druck am Beobachter

$$R_{v_r^2 p'}(\mathbf{x},\mathbf{y},\tau) = \left\langle v_r^2(\mathbf{y},t)\,p'(\mathbf{x},t+\tau)\right\rangle$$

Theoretische Grundlagen

- Korrelationsfunktion $R_{v_r^2 p'}(\mathbf{x}, \mathbf{y}, \tau)$ charakterisiert wieviel die Quellstärke am Ort $\mathbf{y}$ zum beobachteten Schall am Ort $\mathbf{x}$ beiträgt
- Jedoch schwierig zu interpretieren
- Liefert lokale effektive Quellestärke nur falls Quellen räumlich unkorreliert sind
- Erfahrung: v_r', u', v' korrelieren genauso gut wie v_r^2

$$v_r = \bar{v}_r + v_r'$$
$$\left\langle v_r^2\, p' \right\rangle = \bar{v}_r^2 \left\langle p' \right\rangle + 2\bar{v}_r \left\langle \bar{v}_r'\, p' \right\rangle + \left\langle \bar{v}_r'^{\;2}\, p' \right\rangle \approx 2\bar{v}_r \left\langle \bar{v}_r'\, p' \right\rangle$$

Theoretische Grundlagen

Powell/Howe Lighthill's akustische Analogie

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2\,\Delta\right)\rho' = \rho_0\,\mathrm{div}(\boldsymbol{\omega} \times \mathbf{v})$$

$$\rho'(\mathbf{x}, t) \approx -\frac{\rho_0 x_i}{4\pi c_0^3\,|\mathbf{x}|^2} \int\limits_V \frac{\partial(\boldsymbol{\omega} \times \mathbf{v})_i}{\partial t}(\mathbf{y}, t - \tau_0)\,\mathrm{d}^3\mathbf{y}$$

- Approximation $T_{ij} \approx \rho_0 v_i v_j$
- Wirbelstärke $\boldsymbol{\omega}$ wichtige Größe im Quellterm
  - Aus PIV-Messungen eine Komponente von $\boldsymbol{\omega}$ bestimmbar (ω_z)
- PIV-Messung: Korrelation mit u', v' und ω_z' möglich

Spektrum SCCH